2.2 — Random Variables & Distribut

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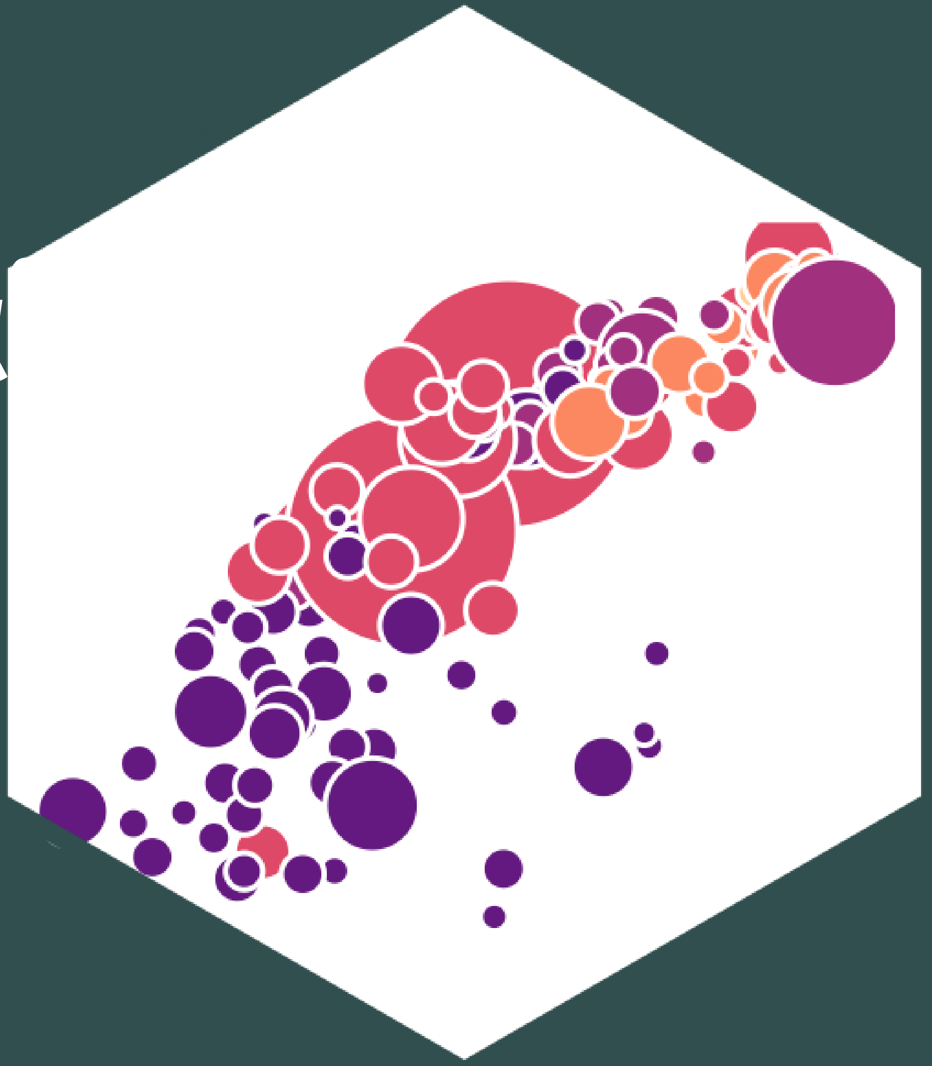


Contents

Random Variables

Experiments

- An **experiment** is any procedure that can (in principle) be repeated infinitely and has a well-defined set of outcomes

> **Example**
>
> Flip a coin 10 times.

Random Variables

- A **random variable (RV)** takes on values that are unknown in advance, but determined by an experiment
- A numerical summary of a random outcome

Example

The number of heads from 10 coin flips

Random Variables: Notation

- Random variable X takes on individual values (x_i) from a set of possible values
- Often capital letters to denote RV's
    - lowercase letters for individual values

Example

Let X be the number of Heads from 10 coin flips. $\quad x_i \in \{0, 1, 2,...,10\}$

Discrete Random Variables

Discrete Random Variables

- A **discrete random variable**: takes on a finite/countable set of possible values

Example

Let X be the number of times your computer crashes this semester[1], $x_i \in \{0, 1, 2, 3, 4\}$

1. Please, back up your files!

Discrete Random Variables: Probability Distribution

- **Probability distribution** of a R.V. fully lists all the possible values of X and their associated probabilities

x_i	$P(X=x_i)$
0	0.80
1	0.10
2	0.06
3	0.03
4	0.01

Discrete Random Variables: pdf

- **Probability distribution function (pdf)** summarizes the possible outcomes of X and their probabilities
- Notation: f_X is the pdf of X:

$$f_X=p_i, \quad i=1,2,...,k$$

- For any real number x_i, $f(x_i)$ is the probablity that $X=x_i$

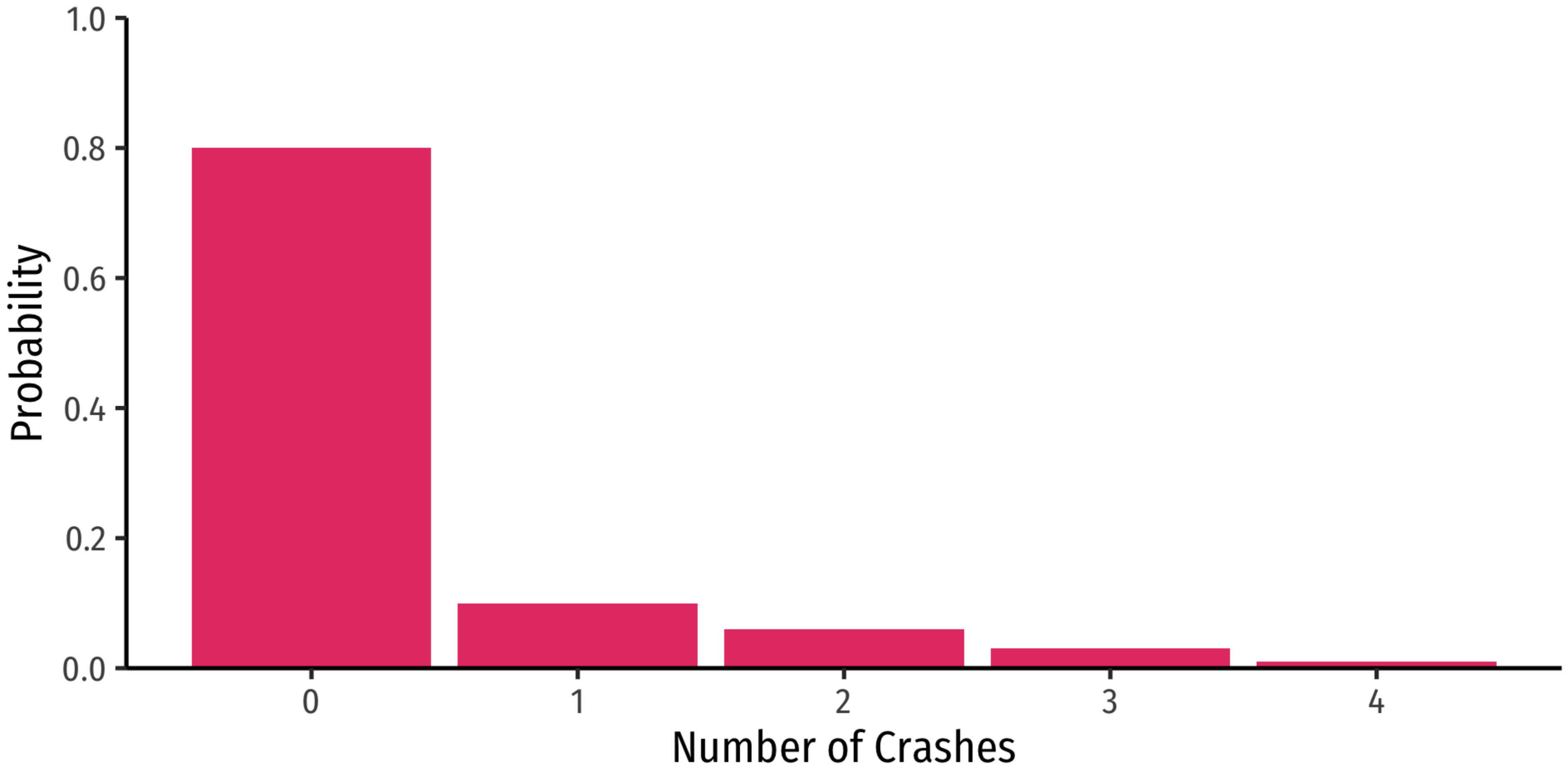
- What is $f(0)$?

- What is $f(3)$?

x_i	$P(X=x_i)$
0	0.80
1	0.10
2	0.06
3	0.03
4	0.01

Discrete Random Variables: pdf Graph

Plot | Code



Discrete Random Variables: cdf

- **Cumulative distribution function (cdf)** lists probability X will be *at most* (less than or equal to) a given value x_i
- Notation: $F_X=P(X \leq x_i)$

x_i	$f(x)$	$F(x)$
0	0.80	0.80
1	0.10	0.90
2	0.06	0.96
3	0.03	0.99
4	0.01	1.00

- What is the probability your computer will crash *at most* once, $F(1)$?
- What about three times, $F(3)$?

Discrete Random Variables: cdf Graph

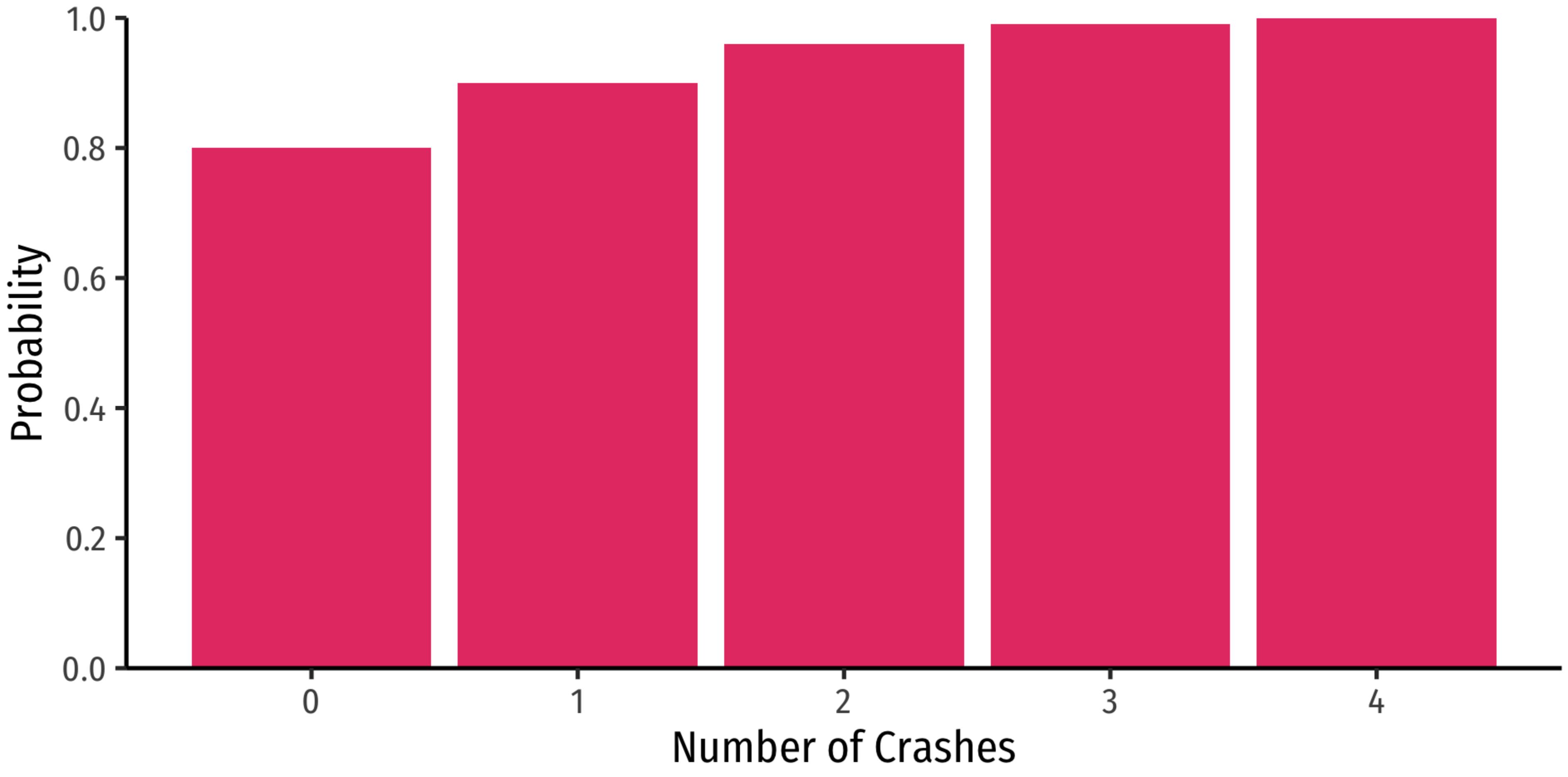
```r
crashes <- crashes %>%
  mutate(cum_prob = cumsum(prob))

crashes
```

```
# A tibble: 5 × 3
  number  prob cum_prob
   <dbl> <dbl>    <dbl>
1      0  0.8      0.8 
2      1  0.1      0.9 
3      2  0.06     0.96
4      3  0.03     0.99
5      4  0.01     1   
```

Discrete Random Variables: cdf Graph

Plot Code



Expected Value and Variance

Expected Value of a Random Variable

- **Expected value** of a random variable X, written $\mathbb{E}(X)$ (and sometimes μ), is the long-run average value of X "expected" after many repetitions

$$\mathbb{E}(X)=\sum^k_{i=1} p_i x_i$$

- $\mathbb{E}(X)=p_1x_1+p_2x_2+ \cdots +p_kx_k$
- A **probability-weighted average** of X, with each x_i weighted by its associated probability p_i
- Also called the **"mean"** or **"expectation"** of X, always denoted either $\mathbb{E}(X)$ or μ_X

Expected Value: Example I

Example

Suppose you lend your friend \$100 at 10% interest. If the loan is repaid, you receive \$110. You estimate that your friend is 99% likely to repay, but there is a default risk of 1% where you get nothing. What is the expected value of repayment?

Expected Value: Example II

Example

Let X be a random variable that is described by the following pdf:

x_i	$P(X=x_i)$
1	0.50
2	0.25
3	0.15
4	0.10

Calculate $\mathbb{E}(X)$.

The Steps to Calculate E(X), Coded

```r
# Make a Random Variable called X
X <- tibble(x_i = c(1,2,3,4), # values of X
            p_i = c(0.50,0.25,0.15,0.10)) # probabilities
```

```r
# Look at tibble
X
```

```
# A tibble: 4 × 2
    x_i   p_i
  <dbl> <dbl>
1     1  0.5 
2     2  0.25
3     3  0.15
4     4  0.1 
```

```r
# Get expected value
X %>%
  summarize(expected_value = sum(x_i * p_i))
```

```
# A tibble: 1 × 1
  expected_value
           <dbl>
1           1.85
```

Variance of a Random Variable

- The **variance** of a random variable X, denoted $\mathbf{var(X)}$ or $\boldsymbol{\sigma^2_X}$ is:

$$\begin{align*}\sigma^2_X &= \mathbb{E}[(x_i-\mu_X)^2]\\ &=\sum_{i=1}^n(x_i-\mu_X)^2p_i\\ \end{align*}$$

- This is the **expected value of the squared deviations from the mean**
  - i.e. the probability-weighted average of the squared deviations

Standard Deviation of a Random Variable

- The **standard deviation** of a random variable X, denoted **$sd(X)$** or **σ_X** is:

$$\sigma_X=\sqrt{\sigma_X^2}$$

- This is the average or expected deviation from the mean

Standard Deviation: Example I

Example

What is the standard deviation of computer crashes?

\(x_i\)	\(P(X=x_i)\)
0	0.80
1	0.10
2	0.06
3	0.03
4	0.01

The Steps to Calculate sd(X), Coded I

```r
# get the expected value
crashes %>%
  summarize(expected_value = sum(number*prob))
```

```
# A tibble: 1 × 1
  expected_value
           <dbl>
1           0.35
```

```r
# save this for quick use
exp_value <- 0.35
```

```r
crashes_2 <- crashes %>%
  select(-cum_prob) %>% # we don't need the cdf
  # create new columns
  mutate(deviations = number - exp_value, # deviations from exp_value
         deviations_sq = deviations^2, # square deviations
         weighted_devs_sq = prob * deviations_sq) # weight squared deviations by probability
```

The Steps to Calculate sd(X), Coded II

```
# look at what we made
crashes_2
```

```
# A tibble: 5 × 5
  number  prob deviations deviations_sq weighted_devs_sq
   <dbl> <dbl>      <dbl>         <dbl>            <dbl>
1      0  0.8       -0.35         0.122           0.098
2      1  0.1        0.65         0.423           0.0423
3      2  0.06       1.65         2.72            0.163
4      3  0.03       2.65         7.02            0.211
5      4  0.01       3.65        13.3             0.133
```

The Steps to Calculate sd(X), Coded III

```r
# now we want to take the expected value of the squared deviations to get variance
crashes_2 %>%
  summarize(variance = sum(weighted_devs_sq), # variance
            sd = sqrt(variance)) # sd is square root of variance
```

```
# A tibble: 1 × 2
  variance    sd
     <dbl> <dbl>
1    0.648 0.805
```

Standard Deviation: Example II

Example

What is the standard deviation of the random variable we saw before?

x_i	$P(X=x_i)$
1	0.50
2	0.25
3	0.15
4	0.10

Hint: you already found it’s expected value.

Continuous Random Variables

Continuous Random Variables

- **Continuous random variables** can take on an uncountable (infinite) number of values
- So many values that the probability of any specific value is infinitely small:

$$P(X=x_i)\rightarrow 0$$

- Instead, we focus on a *range* of values it might take on

Continuous Random Variables: pdf I

- **Probability *density* function (pdf)** of a continuous variable represents the probability between two values as the area under a curve
- The total area under the curve is 1
- Since $P(a)=0$ and $P(b)=0$, $P(a<X<b)=P(a \leq X \leq b)$
- See today's appendix for how to graph math/stats functions in `ggplot`!

Example

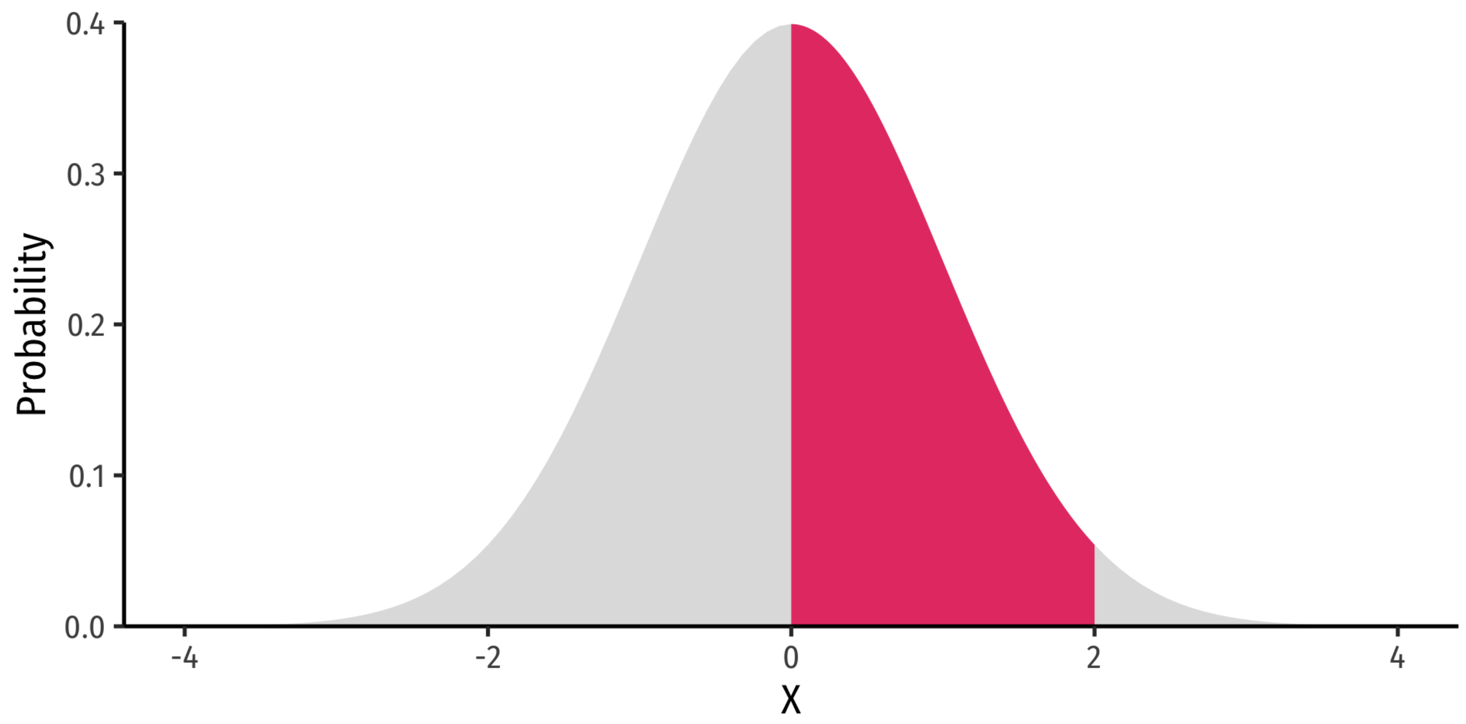
$P(0 \leq X \leq 2)$

Continuous Random Variables: pdf II

- FYI using calculus:

$$P(a \leq X \leq b) = \int_a^b f(x) dx$$

- Complicated: software or (old fashioned!) probability tables to calculate



Example

$P(0 \leq X \leq 2)$

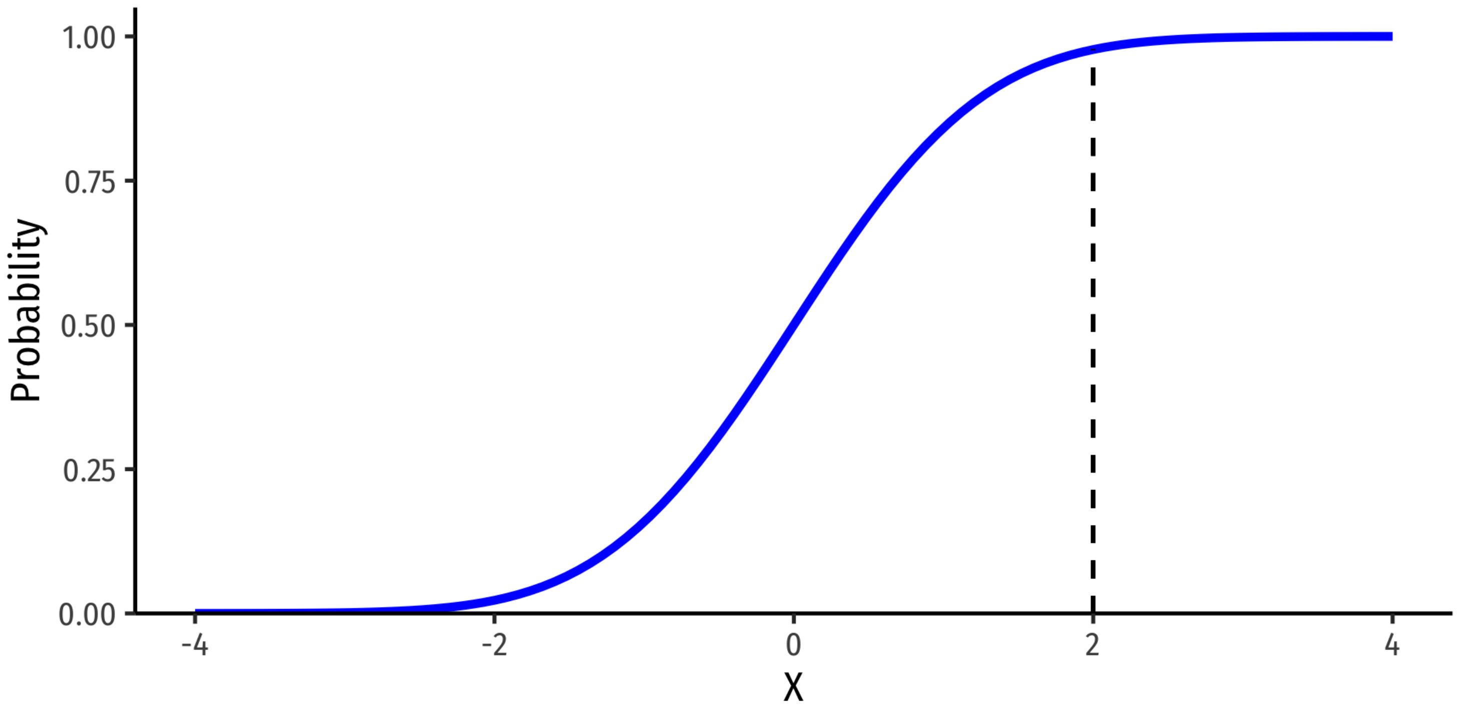
Continuous Random Variables: cdf I

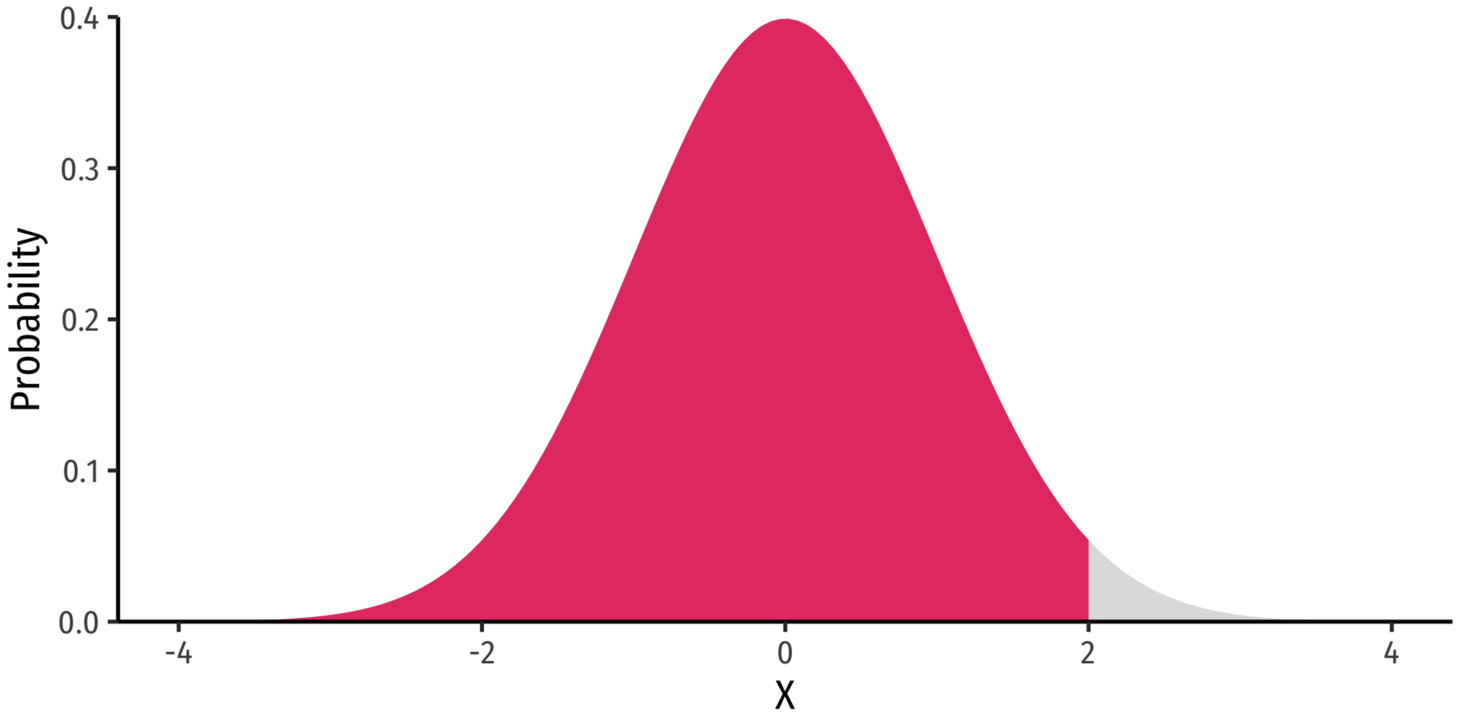
- The **cumulative density function (cdf)** describes the area under the pdf for all values less than or equal to (i.e. to the left of) a given value, k

$$P(X \leq k)$$

Example

$P(X \leq 2)$





Continuous Random Variables: cdf II

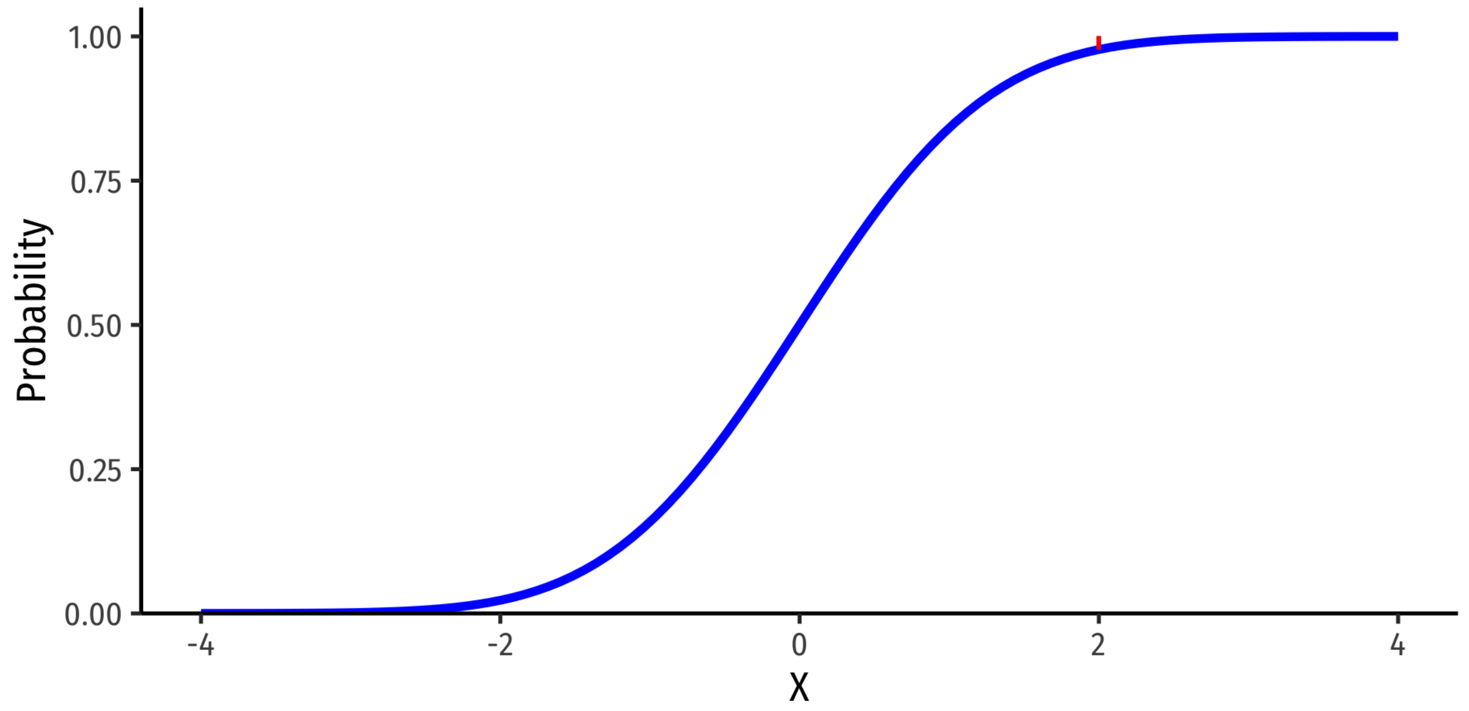
- Note: to find probability of values *greater* than or equal to (to the right of) a given value k:

$$P(X \geq k)=1-P(X \leq k)$$

Example

$P(X \geq 2) = 1 - P(X \leq 2)$

$P(X \geq 2)=$ area under the pdf curve to the right of 2



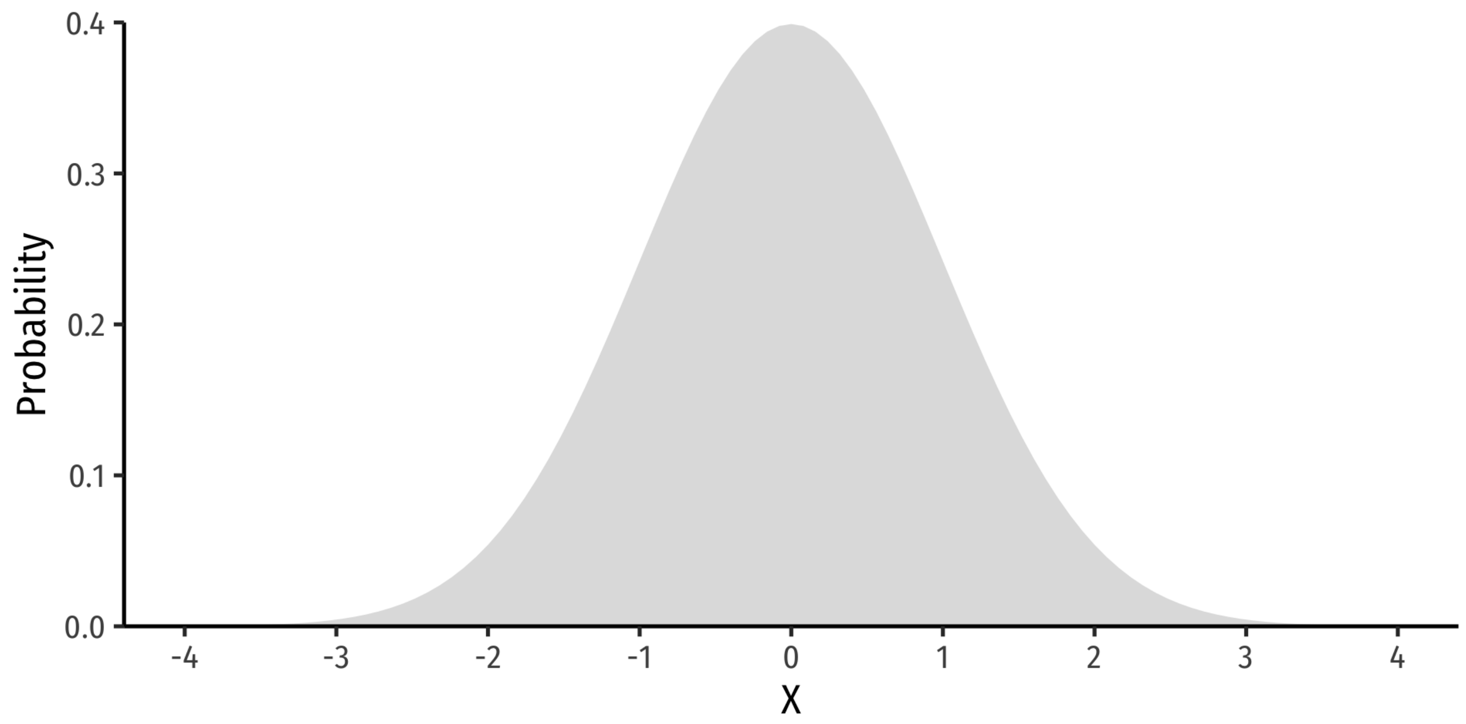
The Normal Distribution

The Normal Distribution

- The **Gaussian** or **normal distribution** is the most useful type of probability distribution

$$X \sim N(\mu,\sigma)$$

- "X is distributed **N**ormally with mean μ and standard deviation σ"
- Continuous, symmetric, unimodal

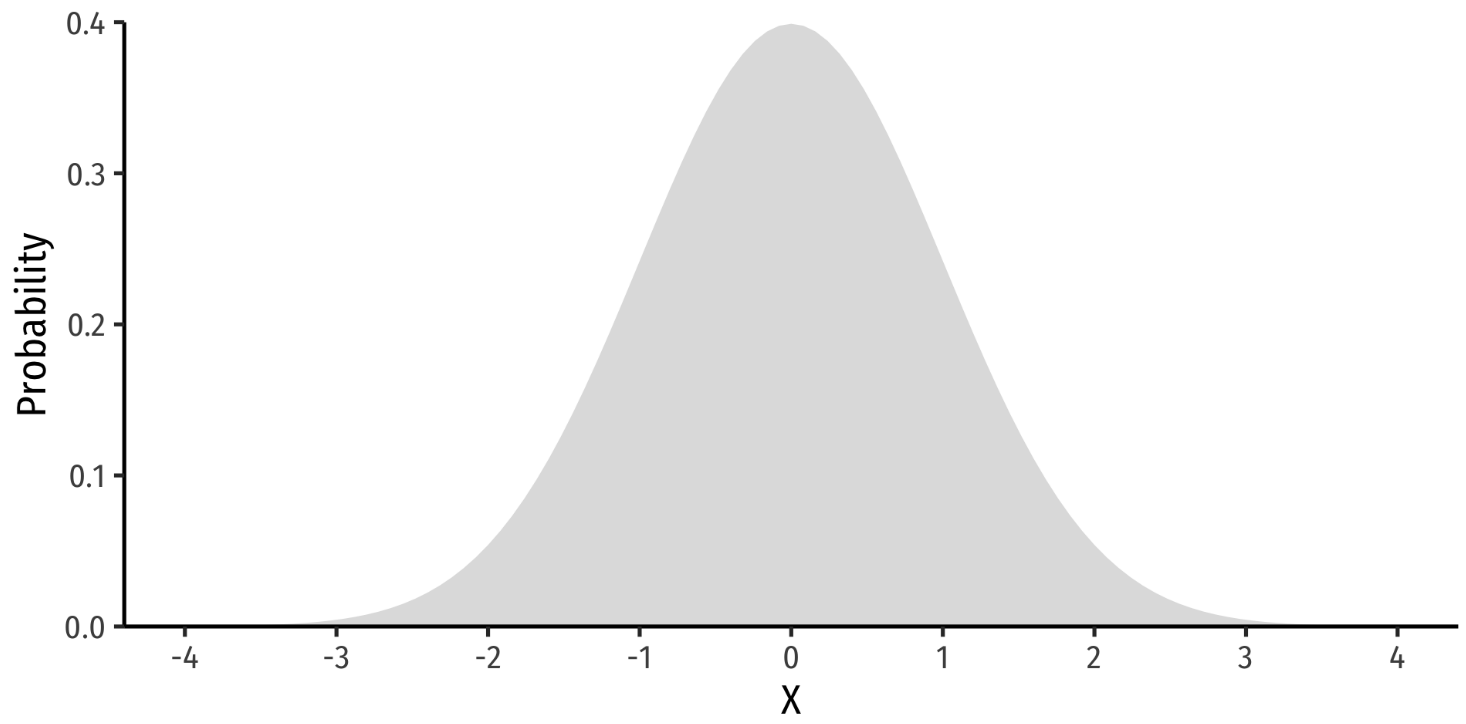


The Normal Distribution: pdf

- FYI: The pdf of $X \sim N(\mu, \sigma)$ is

$$P(X=k)= \frac{1}{\sqrt{2\pi \sigma^2}}e^{-\frac{1}{2}\big(\frac{(k-\mu)}{\sigma}\big)^2}$$

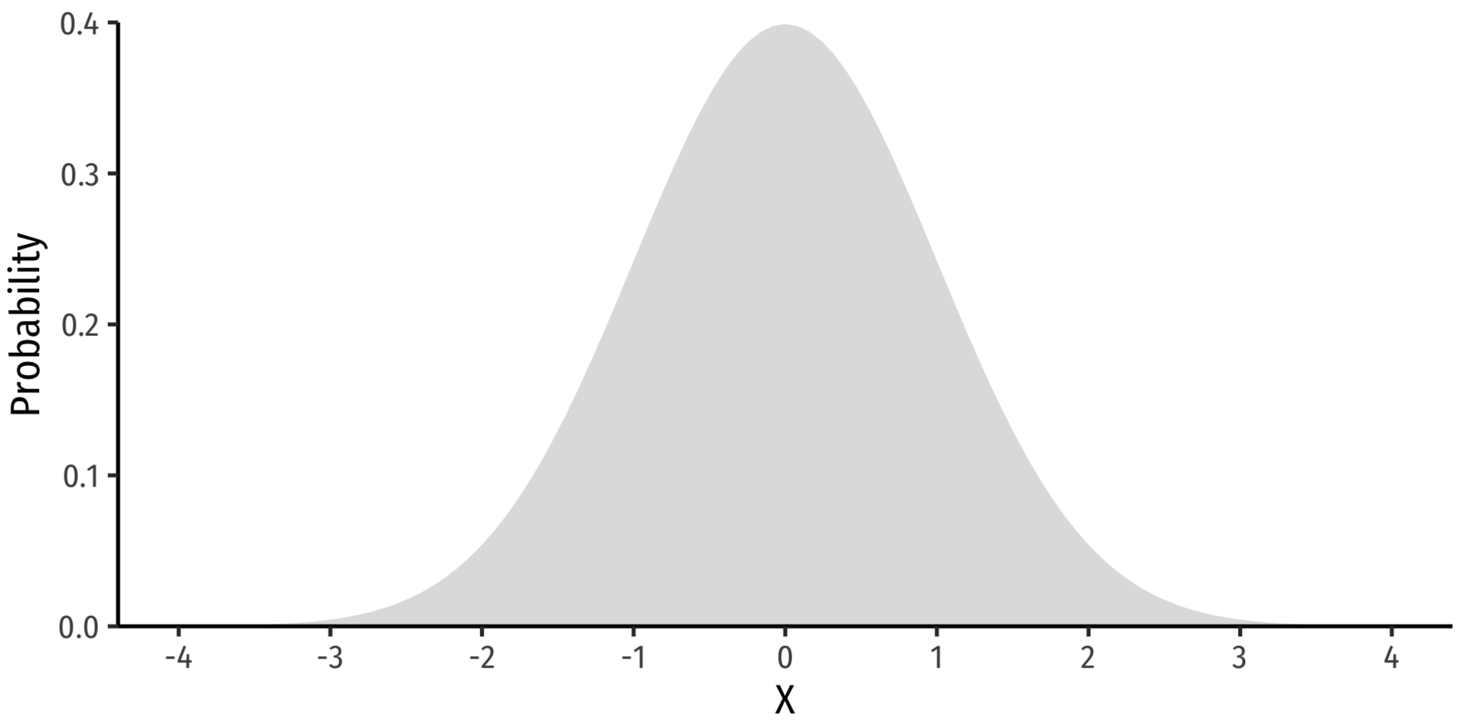
- **Do not try and learn this**, we have software and (previously tables) to calculate pdfs and cdfs



The Standard Normal Distribution

- The **standard** normal distribution (often referred to as Z) has mean 0 and standard deviation 1

$$Z \sim N(0,1)$$

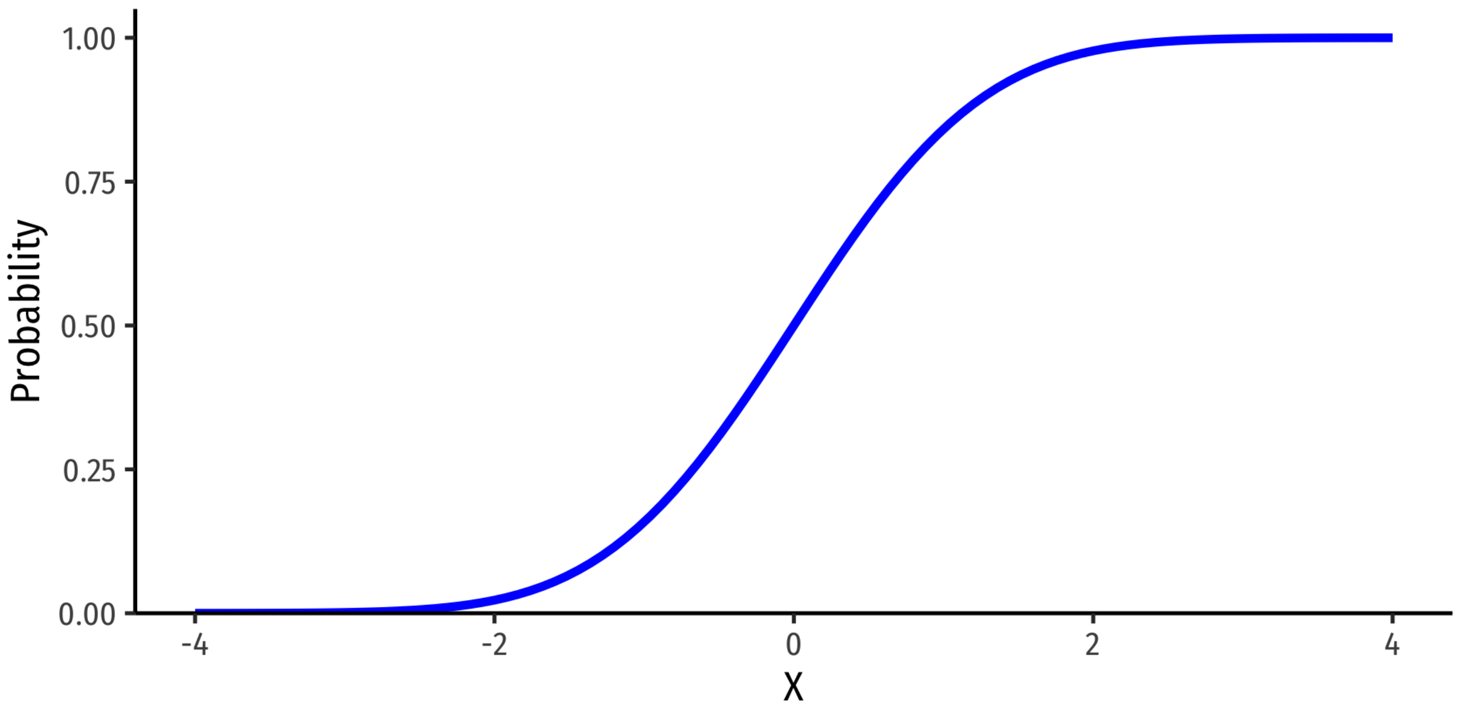


The Standard Normal cdf

- The **standard** normal cdf, often referred to as Φ:

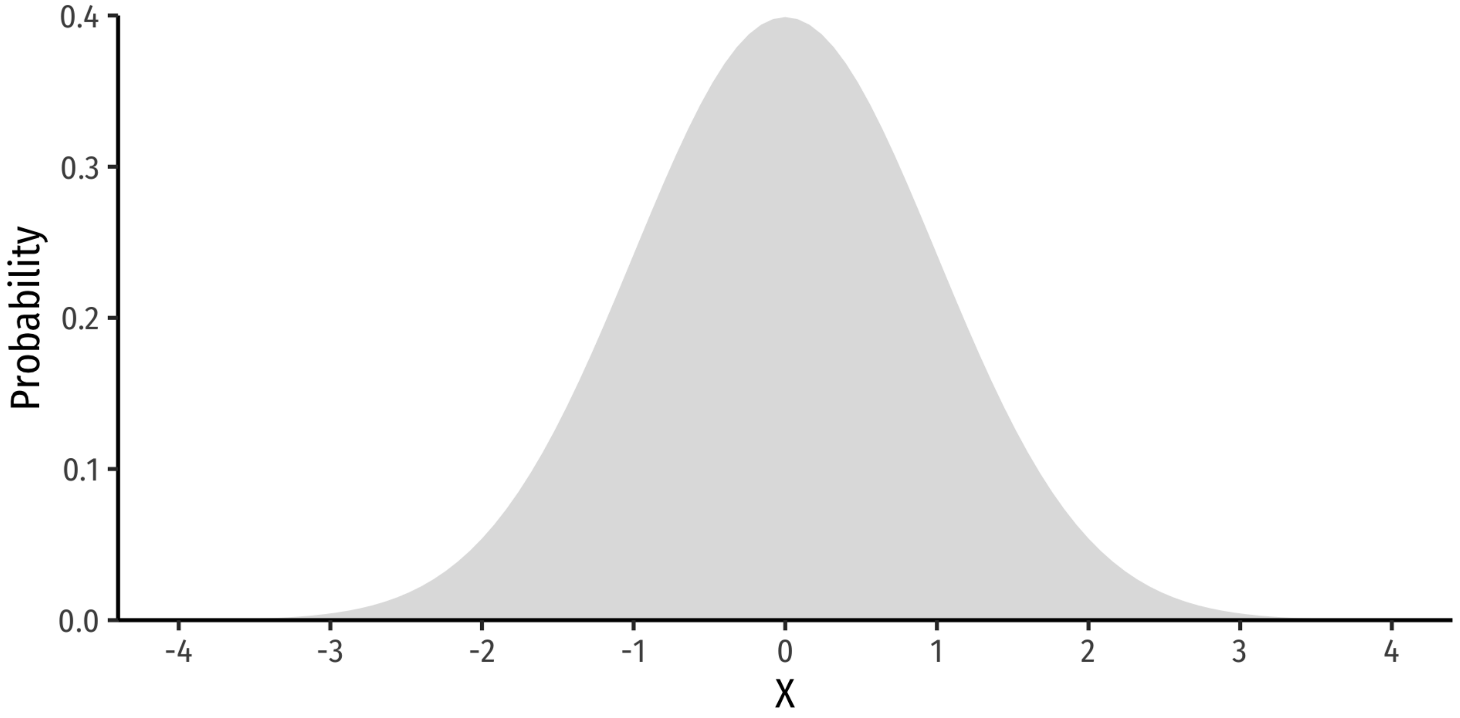
$$\Phi(k)=P(Z \leq k)$$

(again, the area under the pdf curve to the left of some value k)



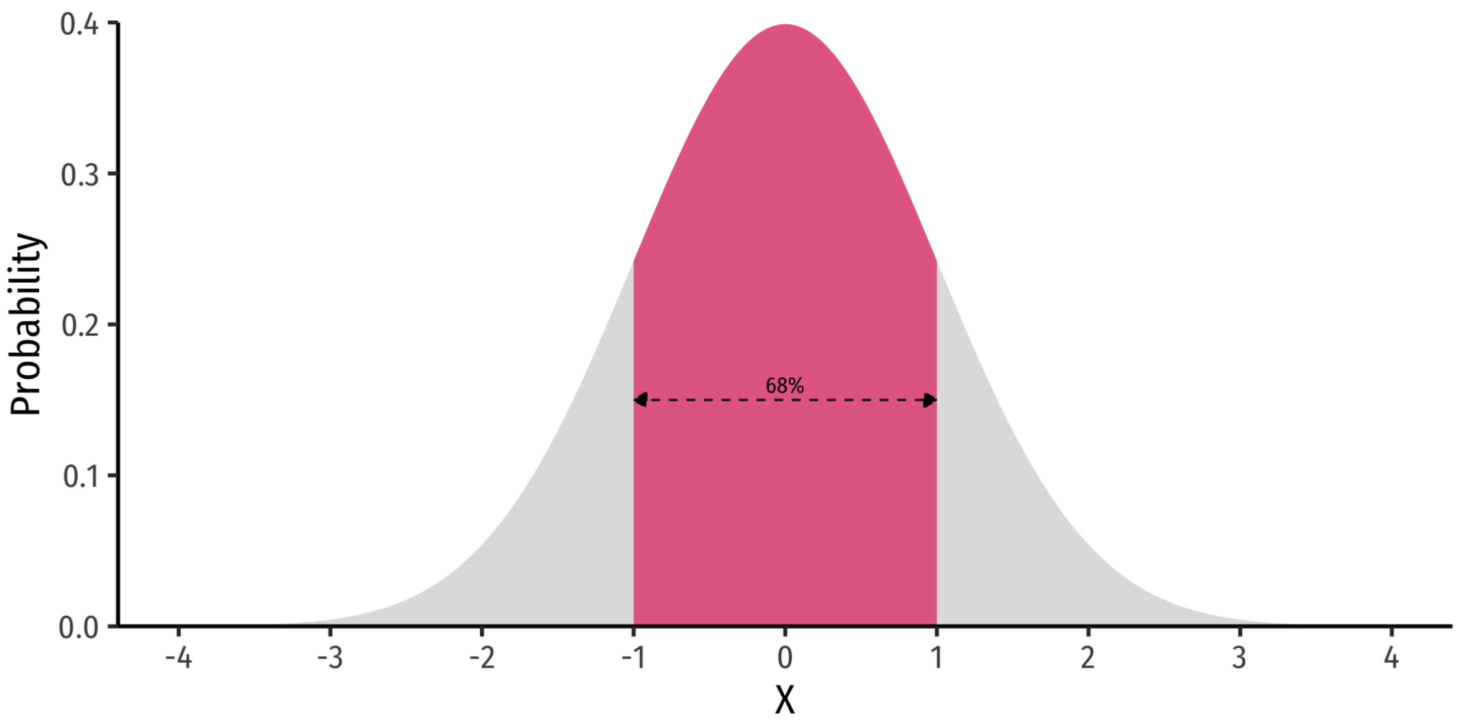
The 68-95-99.7 Empirical Rule

- **68-95-99.7% empirical rule**: for a normal distribution:



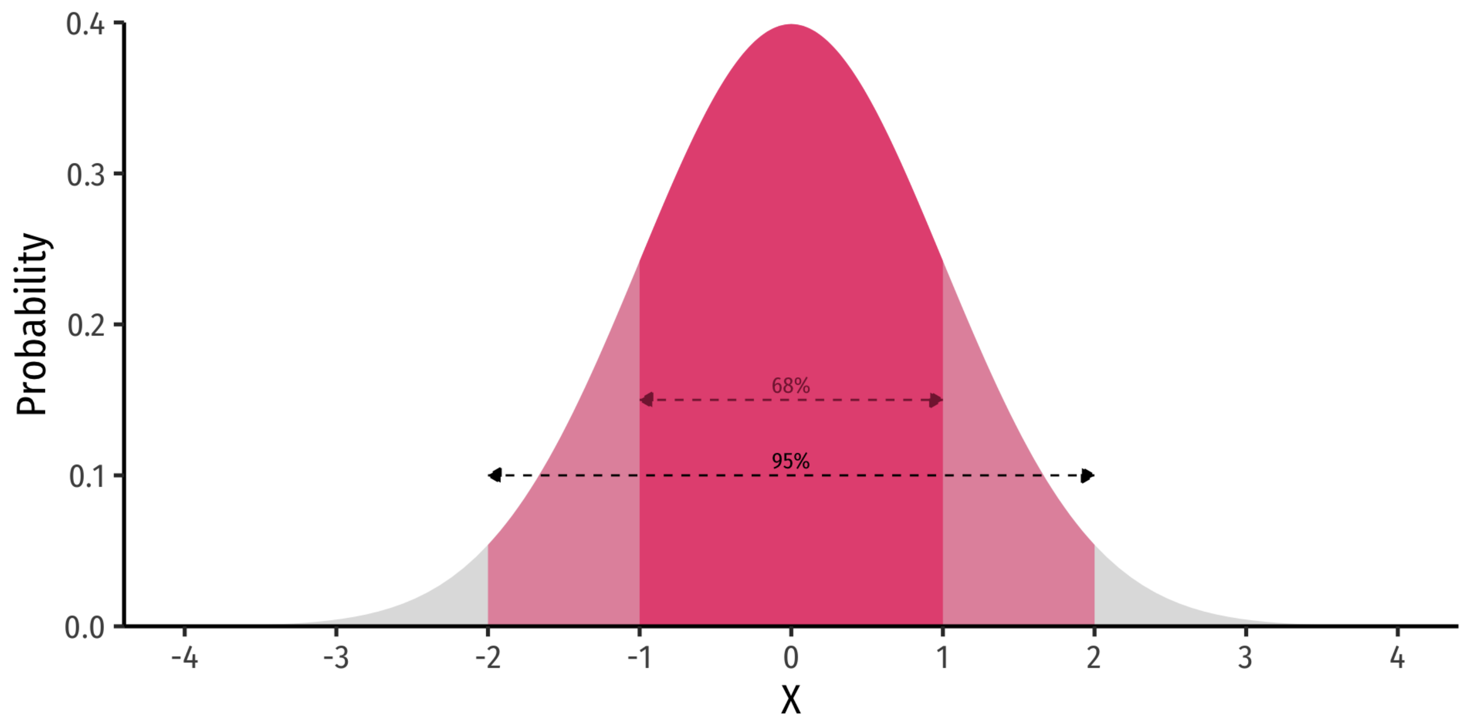
The 68-95-99.7 Empirical Rule

- **68-95-99.7% empirical rule**: for a normal distribution:
- \(P(\mu-1\sigma \leq X \leq \mu+1\sigma) \approx\) 68%



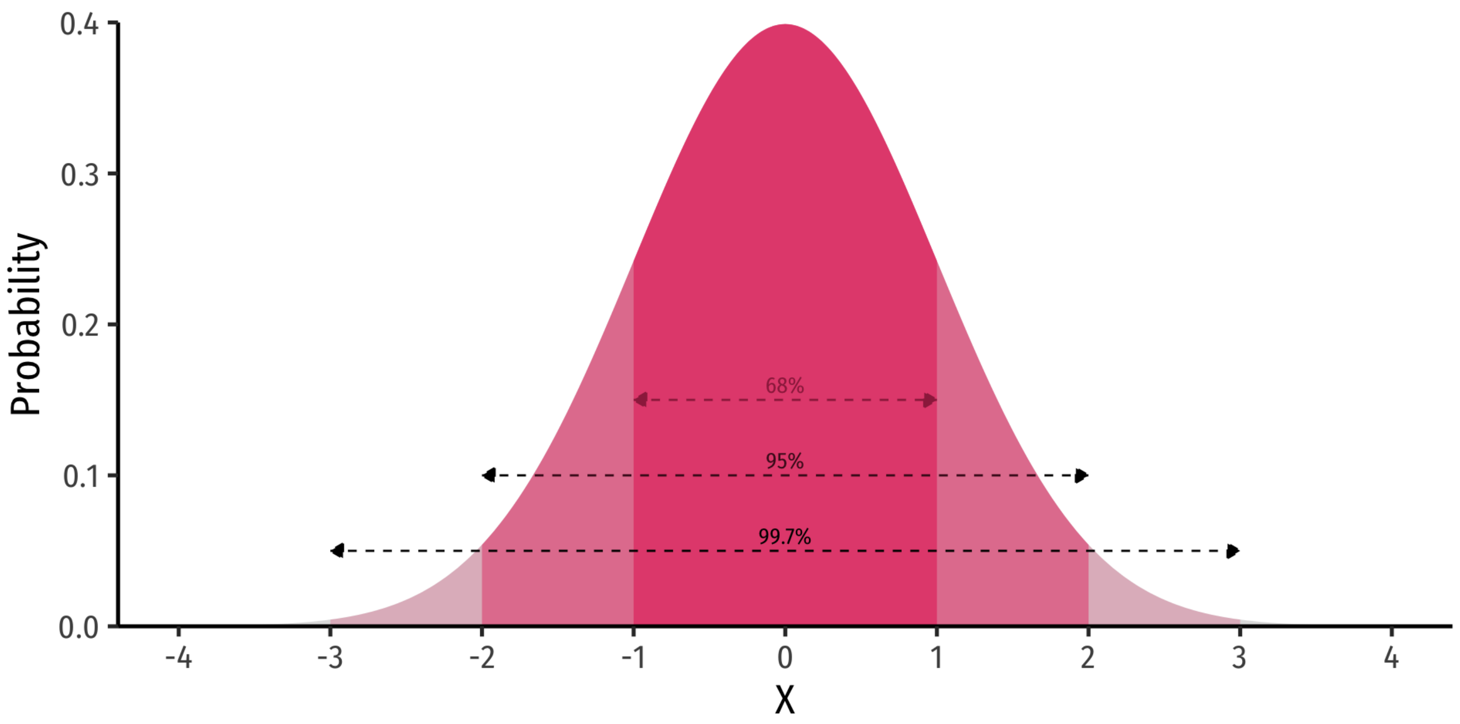
The 68-95-99.7 Empirical Rule

- **68-95-99.7% empirical rule**: for a normal distribution:
- $P(\mu-1\sigma \leq X \leq \mu+1\sigma) \approx$ 68%
- $P(\mu-2\sigma \leq X \leq \mu+2\sigma) \approx$ 95%



The 68-95-99.7 Empirical Rule

- **68-95-99.7% empirical rule**: for a normal distribution:
- $P(\mu-1\sigma \leq X \leq \mu+1\sigma) \approx$ 68%
- $P(\mu-2\sigma \leq X \leq \mu+2\sigma) \approx$ 95%
- $P(\mu-3\sigma \leq X \leq \mu+3\sigma) \approx$ 99.7%
- **68/95/99.7%** of observations fall within **1/2/3 standard deviations** of the mean

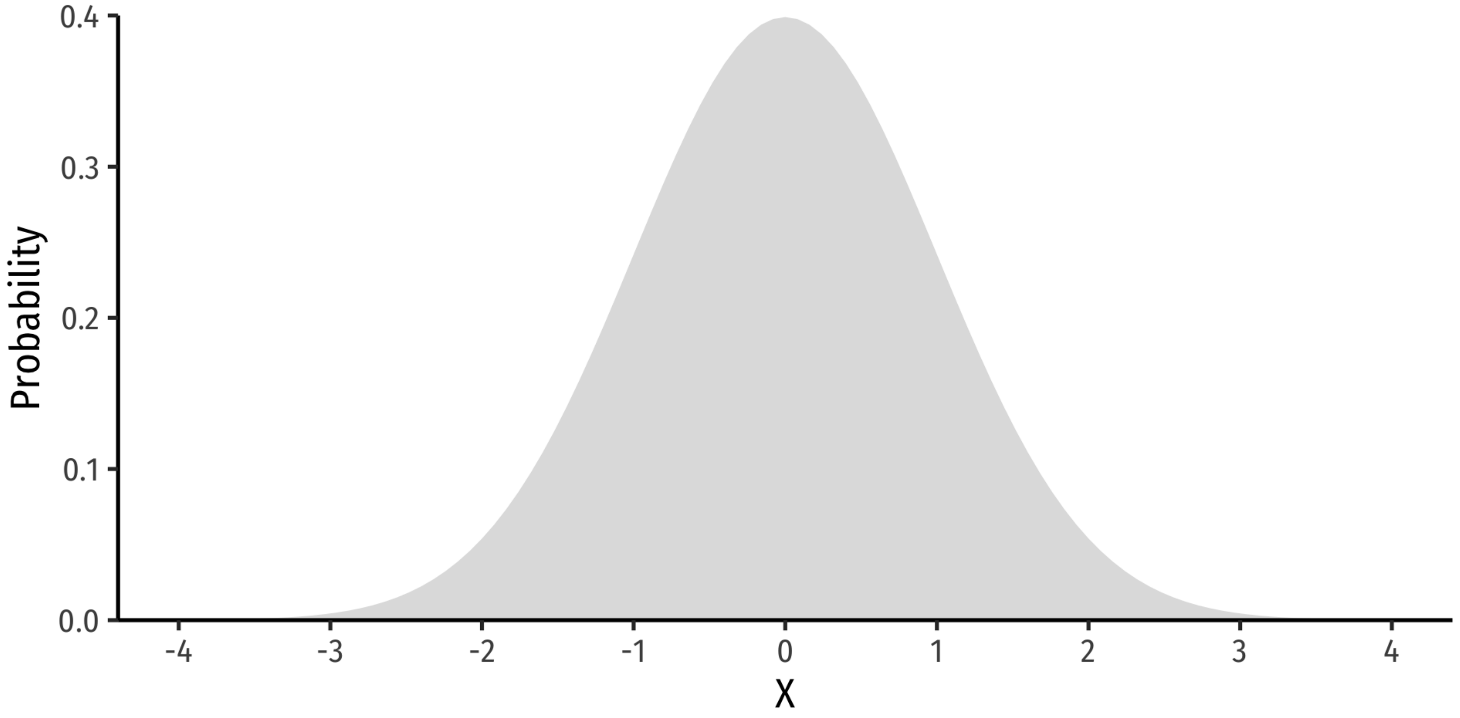


Standardizing Normal Distributions

- We can take any normal distribution (for any (μ, σ)) and **standardize** it to the standard normal distribution by taking the **Z-score** of any value, x_i:

$$Z=\frac{x_i-\mu}{\sigma}$$

- Subtract any value by the distribution's mean and divide by standard deviation
- Z: number of standard deviations x_i value is away from the mean



Standardizing Normal Distributions: Example I

Example

On August 8, 2011, the Dow dropped 634.8 points, sending shock waves through the financial community. Assume that during mid-2011 to mid-2012 the daily change for the Dow is normally distributed, with the mean daily change of 1.87 points and a standard deviation of 155.28 points. What is the Z-score?

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{634.8-1.87}{155.28}$$

$$Z = -4.1$$

This is 4.1 standard deviations (σ) beneath the mean, an *extremely* low probability event.

Standardizing Normal Distributions: Example II

Example

In the last quarter of 2021, a group of 64 mutual funds had a mean return of 2.4% with a standard deviation of 5.6%. These returns can be approximated by a normal distribution.

What percent of the funds would you expect to be earning between -3.2% and 8.0% returns?

Convert to standard normal to find Z-scores for 8 and -3.2.

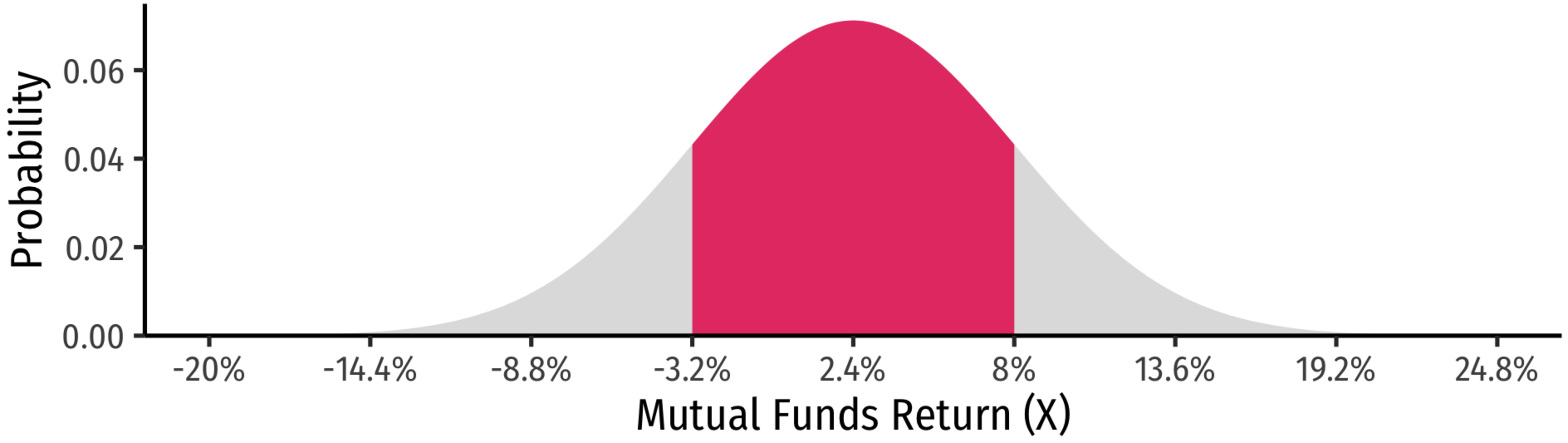
$$P(-3.2 < X < 8)$$

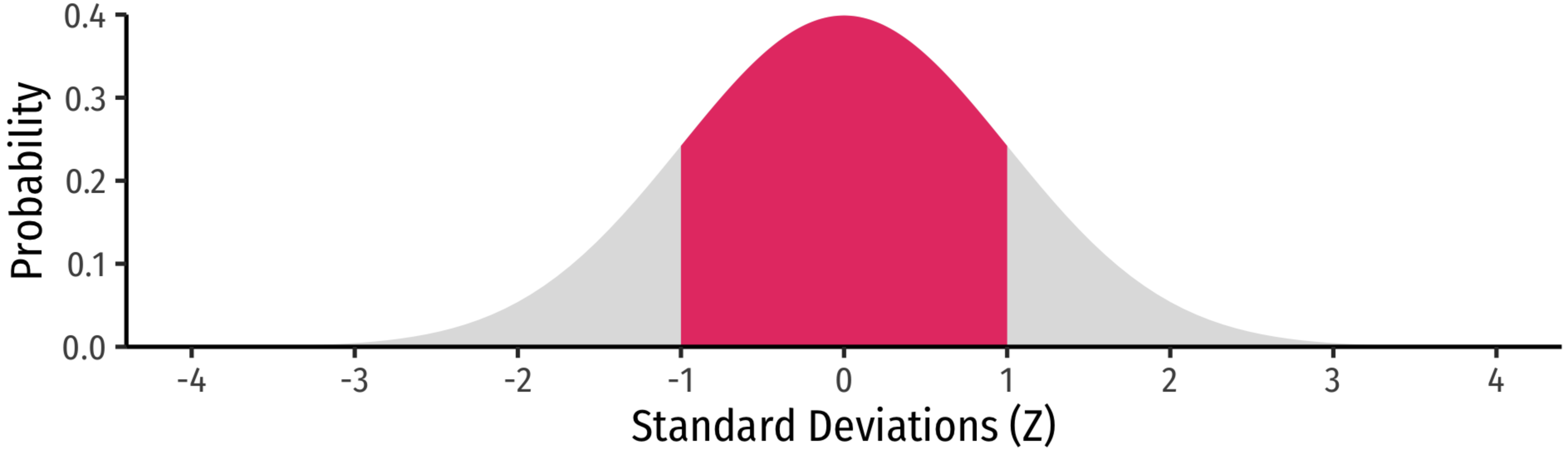
$$P(\frac{-3.2-2.4}{5.6} < \frac{X-2.4}{5.6} < \frac{8-2.4}{5.6})$$

$$P(-1 < Z < 1)$$

$$P(X \pm 1\sigma)=0.68$$

Standardizing Normal Distributions: Example II





Standardizing Normal Distributions: Example III

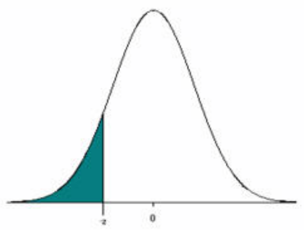
Example

In the last quarter of 2015, a group of 64 mutual funds had a mean return of 2.4% with a standard deviation of 5.6%. These returns can be approximated by a normal distribution.

1. What percent of the funds would you expect to be earning between -3.2% and 8.0% returns?
2. What percent of the funds would you expect to be earning 2.4% or less?
3. What percent of the funds would you expect to be earning between -8.8% and 13.6%?
4. What percent of the funds would you expect to be earning returns greater than 13.6%?

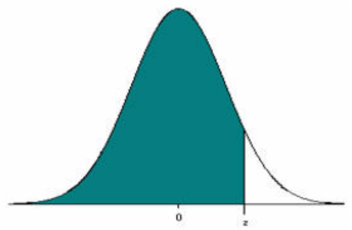
How do we *actually* find the probabilities for Z-scores?

Table of Standard Normal Probabilities for Negative Z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table of Standard Normal Probabilities for Positive Z-scores



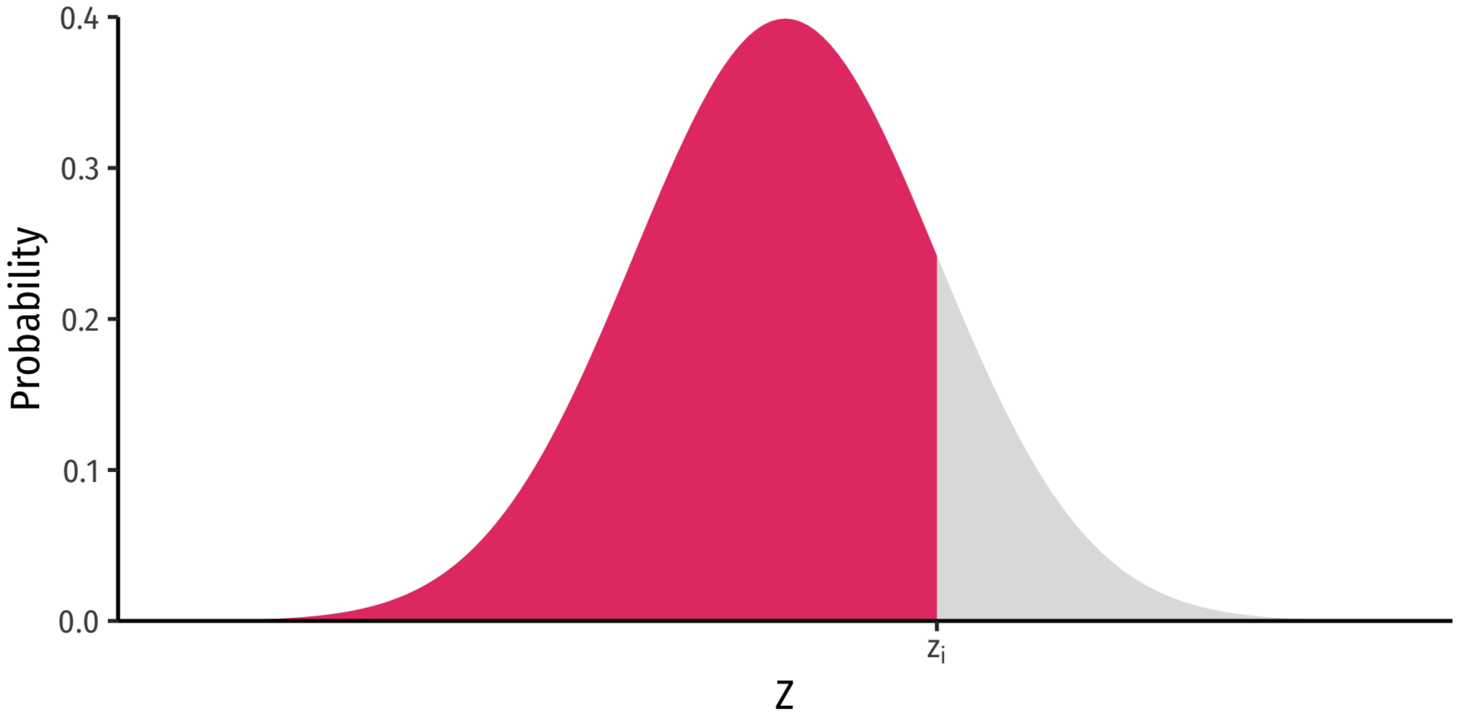
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Note that the probabilities given in this table represent the area to the LEFT of the z-score.
The area to the RIGHT of a z-score = 1 – the area to the LEFT of the z-score

Finding Z-score Probabilities I

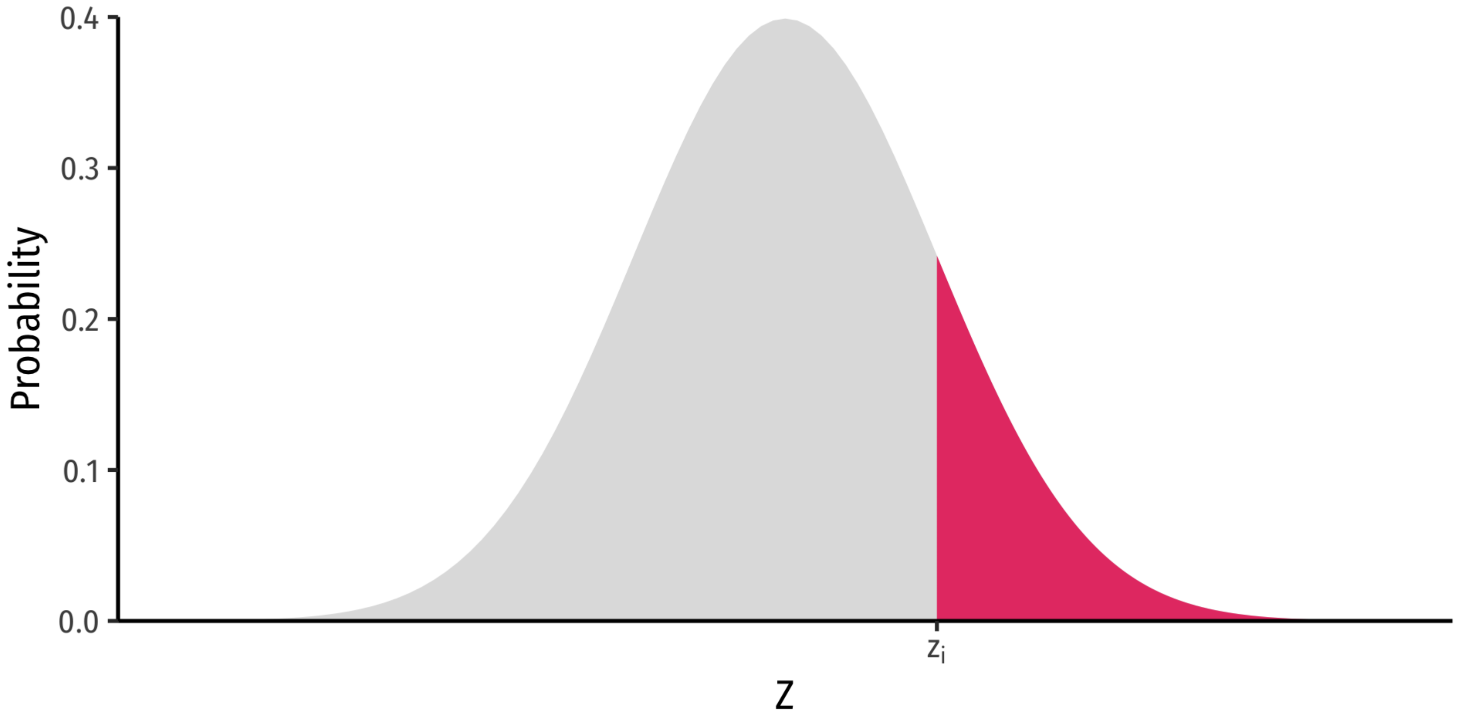
Probability to the **left** of z_i

$$P(Z \leq z_i)= \underbrace{\Phi(z_i)}_{\text{cdf of }z_i}$$



Probability to the **right** of z_i

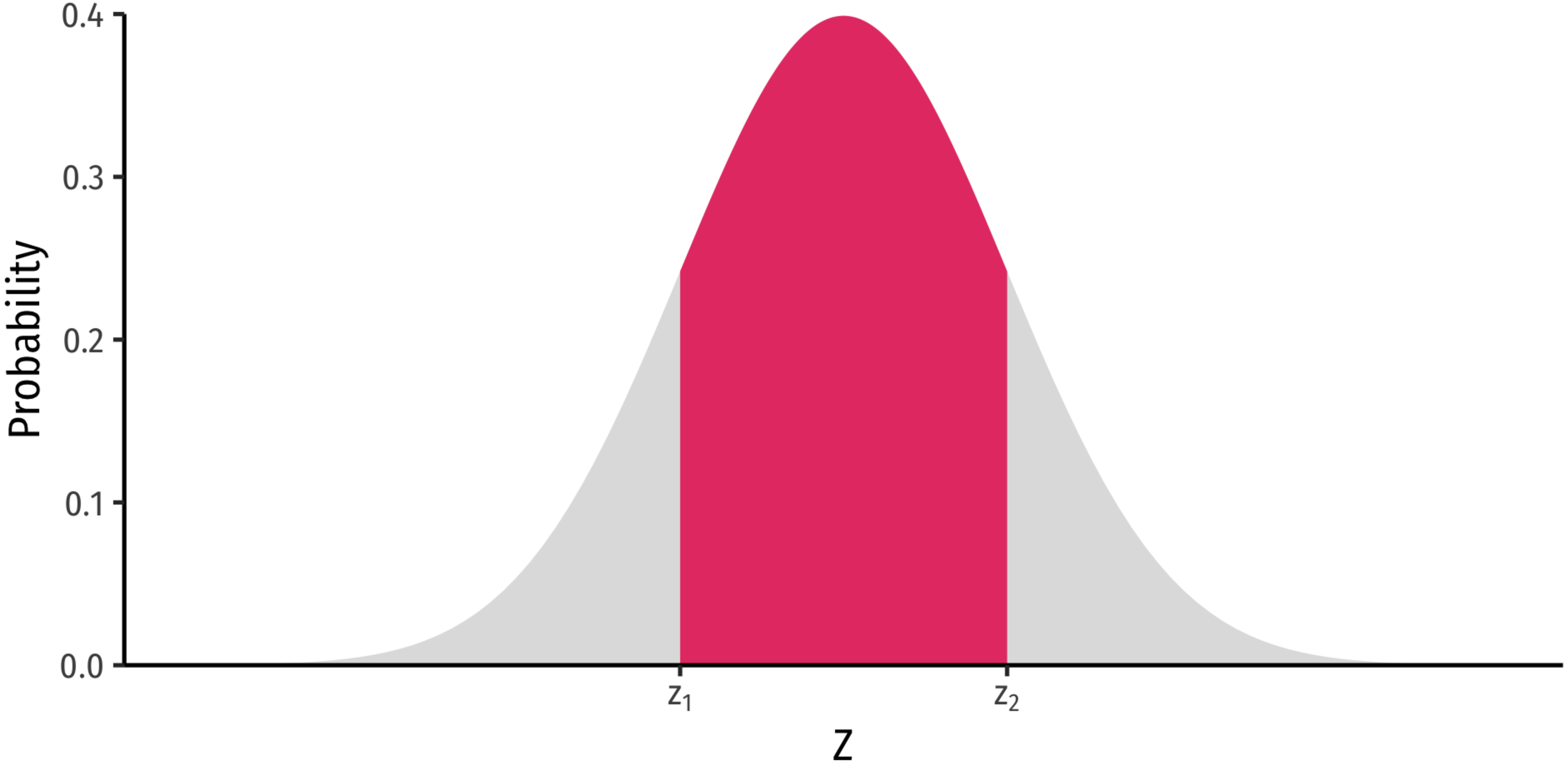
$$P(Z \geq z_i)= 1-\underbrace{\Phi(z_i)}_{\text{cdf of }z_i}$$



Finding Z-score Probabilities II

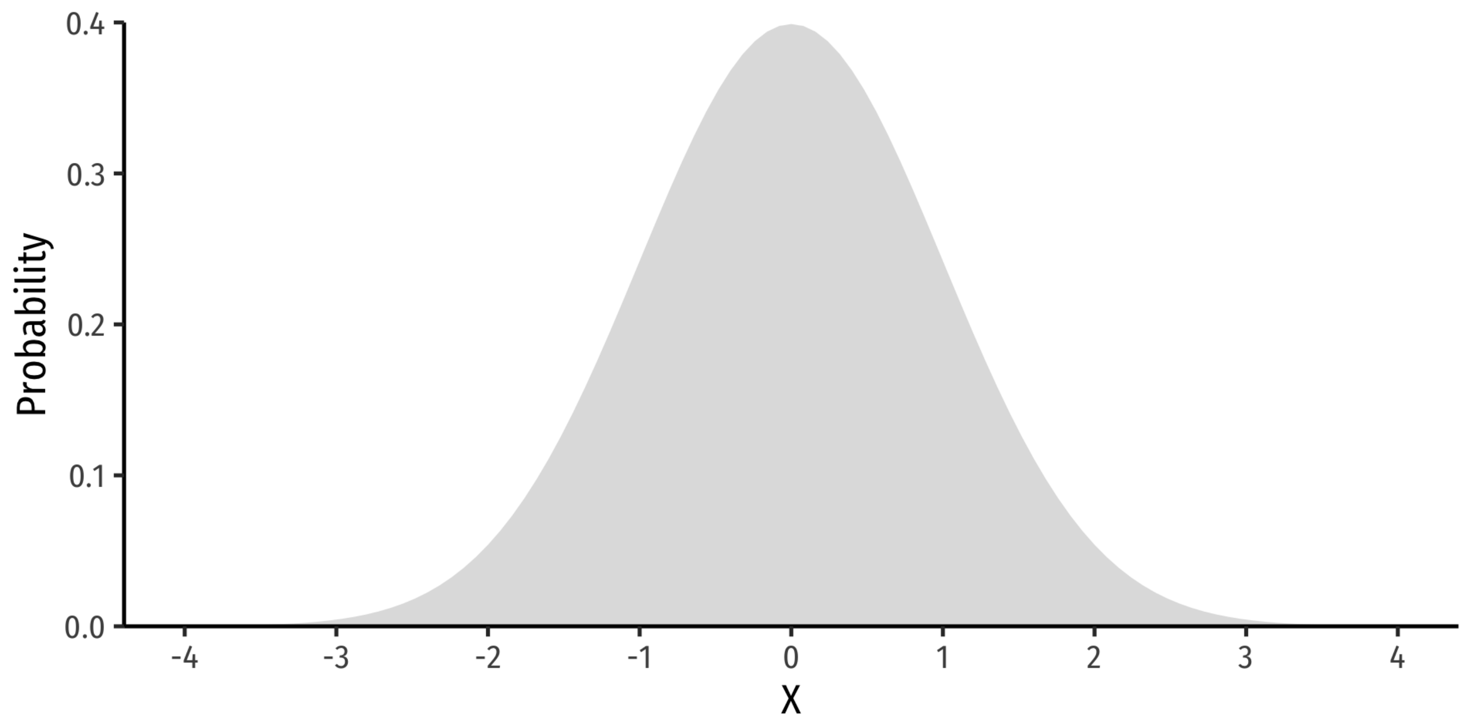
Probability **between** z_1 and z_2

$$P(z_1 \geq Z \geq z_2)= \underbrace{\Phi(z_2)}_{\text{cdf of }z_2} - \underbrace{\Phi(z_1)}_{\text{cdf of }z_1}$$



Finding Z-score Probabilities III

- `pnorm()` calculates probabilities with a normal distribution with arguments:
  - `x =` the value
  - `mean =` the mean
  - `sd =` the standard deviation
  - `lower.tail =`
    - `TRUE` if looking at area to *LEFT* of value
    - `FALSE` if looking at area to *RIGHT* of value



Finding Z-score Probabilities IV

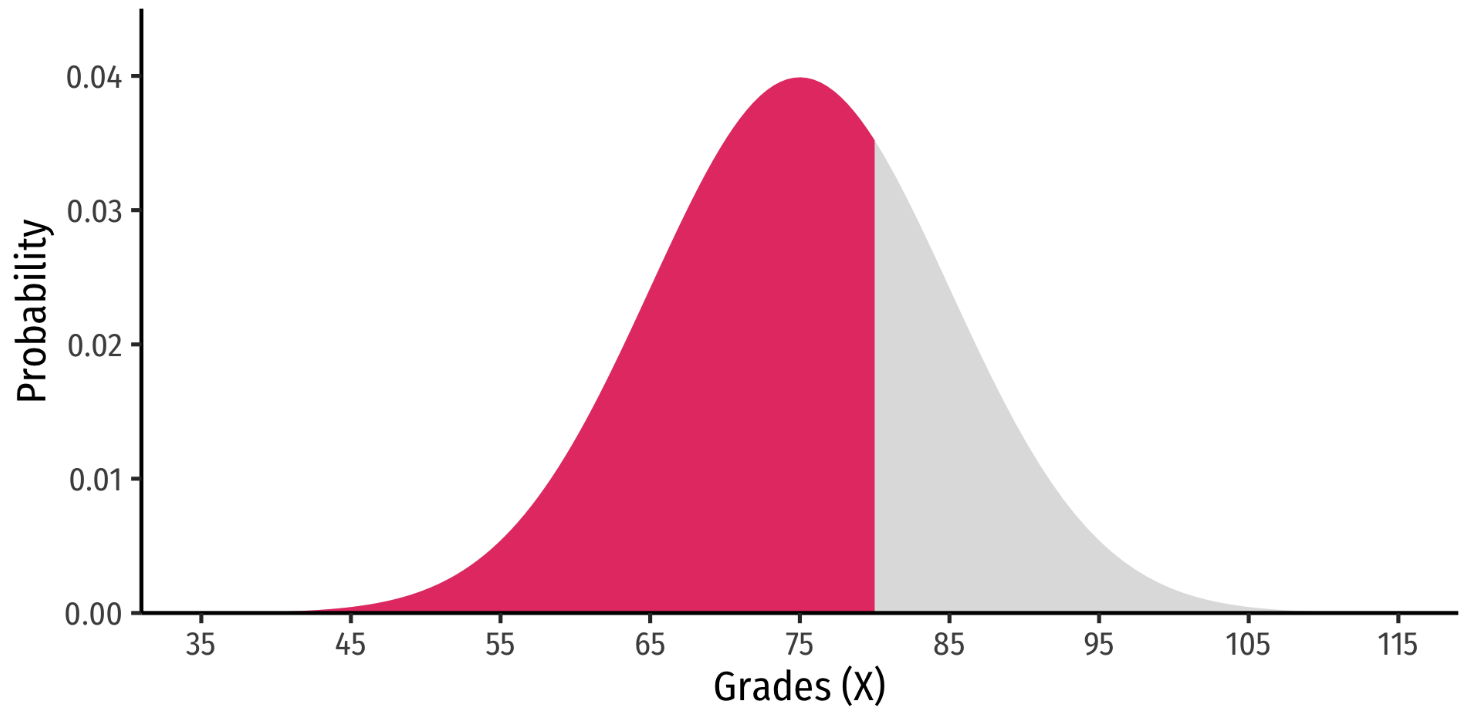
Example

Let the distribution of grades be normal, with mean 75 and standard deviation 10.

- Probability a student gets **at least an 80**

```r
pnorm(80,
      mean = 75,
      sd = 10,
      lower.tail = FALSE) # looking to right
```

```
[1] 0.3085375
```



Finding Z-score Probabilities V

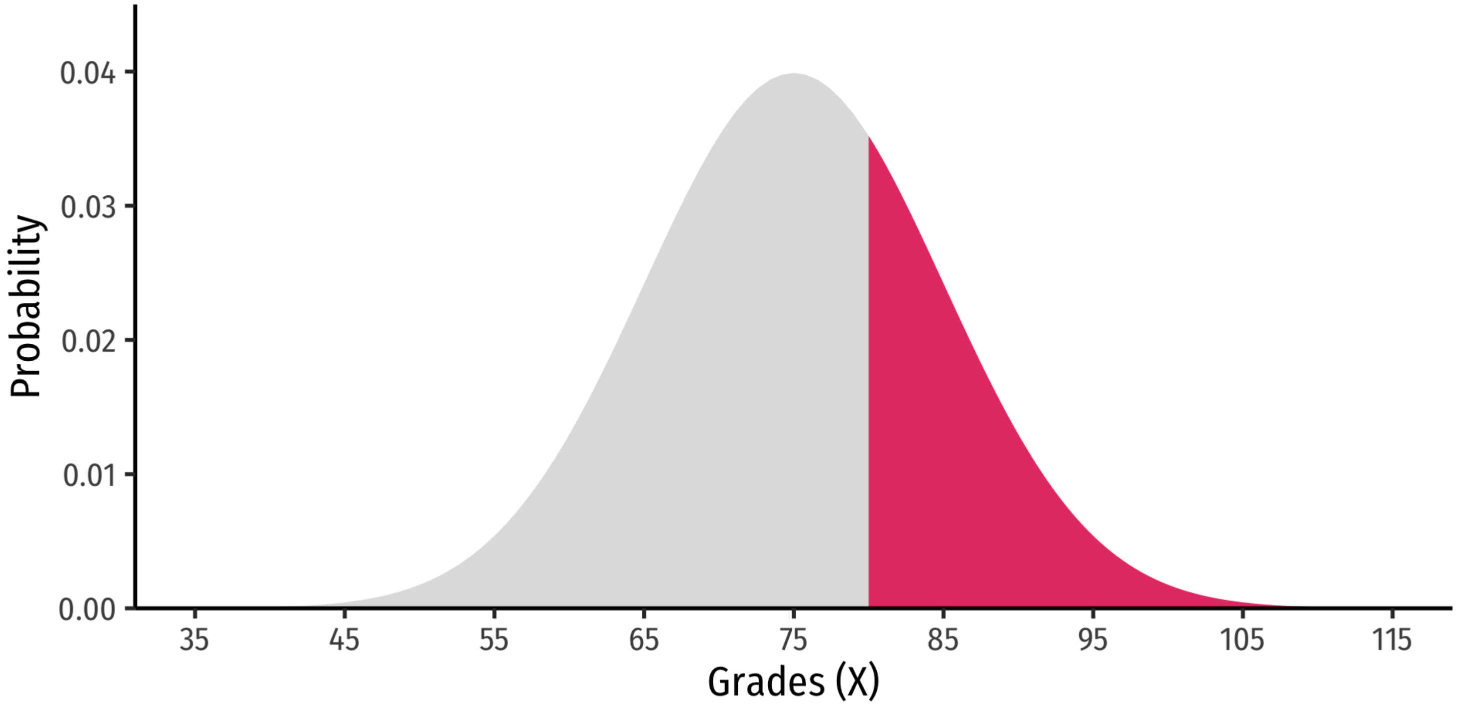
Example

Let the distribution of grades be normal, with mean 75 and standard deviation 10.

- Probability a student gets **at most an 80**

```r
pnorm(80,
      mean = 75,
      sd = 10,
      lower.tail = TRUE) # looking to left
```

```
[1] 0.6914625
```



Finding Z-score Probabilities VI

Example

Let the distribution of grades be normal, with mean 75 and standard deviation 10.

- Probability a student gets **between 65 and 85**

```r
# subtract two left tails!
pnorm(85, # larger number first!
      mean = 75,
      sd = 10,
      lower.tail = TRUE) - # looking to left, & SU
  pnorm(65, # smaller number second!
        mean = 75,
        sd = 10,
        lower.tail = TRUE) #looking to left
```

```
[1] 0.6826895
```