5.3 — Instrumental Variables

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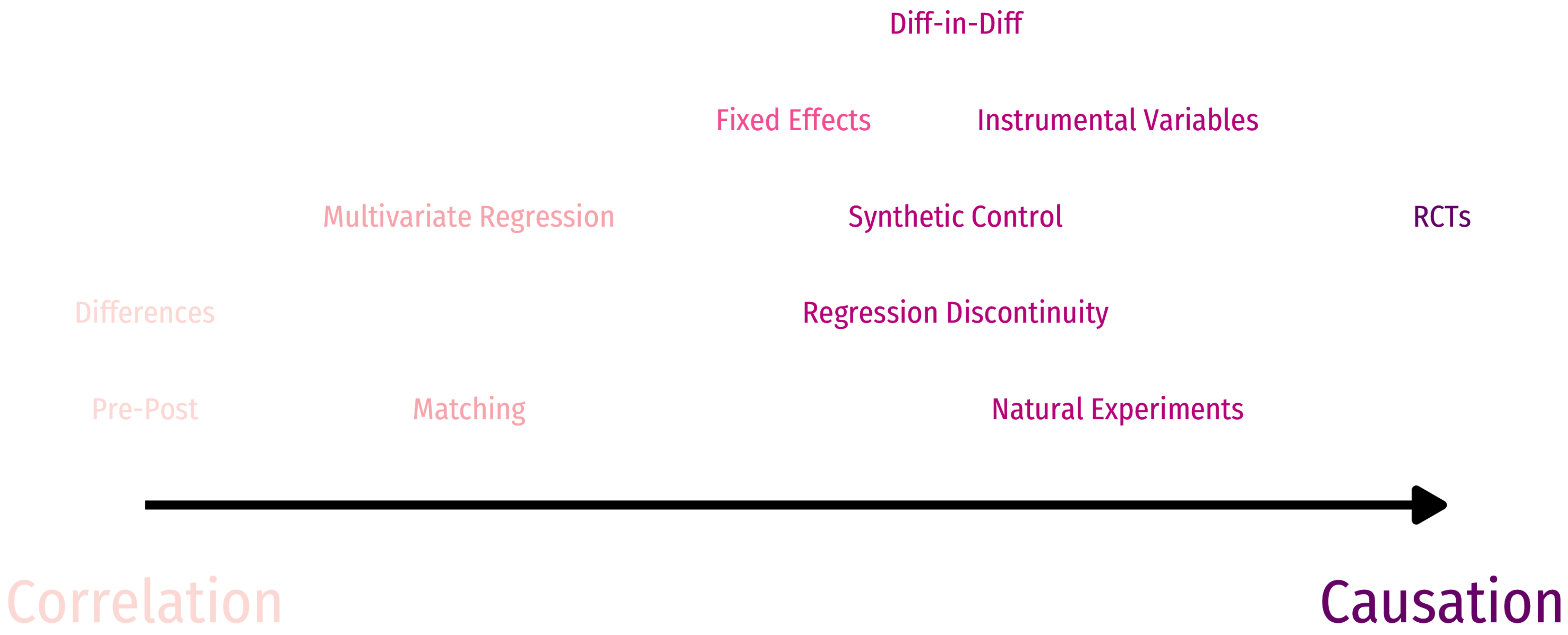
Contents

Clever Research Designs Identify Causality

Again, **this toolkit** of research designs to **identify causal effects** is the economist's **comparative advantage** that firms and governments want!



Identification Strategies

- **Endogeneity** remains the hardest (and most common) econometric challenge
- Diff-n-diff/fixed effects are one strategy to minimize endogeneity
  - *Requires* panel data
  - Can't use time-varying omitted variables that are correlated with regressors
- Another strategy to is to find some source of exogenous variation that removes the endogeneity of a variable, using that source as a **instrumental variable**

Identification Strategies

- **Endogeneity** remains the hardest (and most common) econometric challenge
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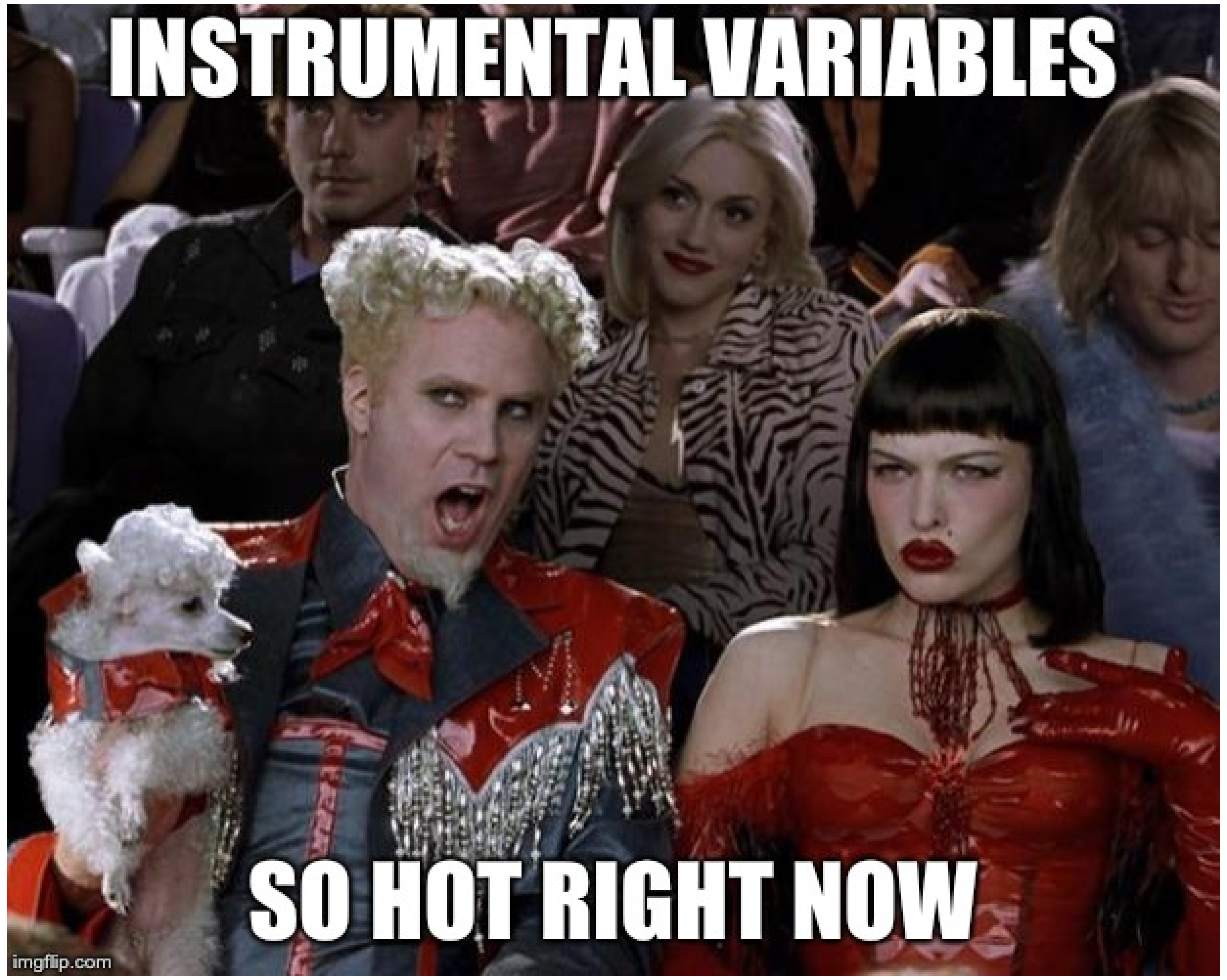
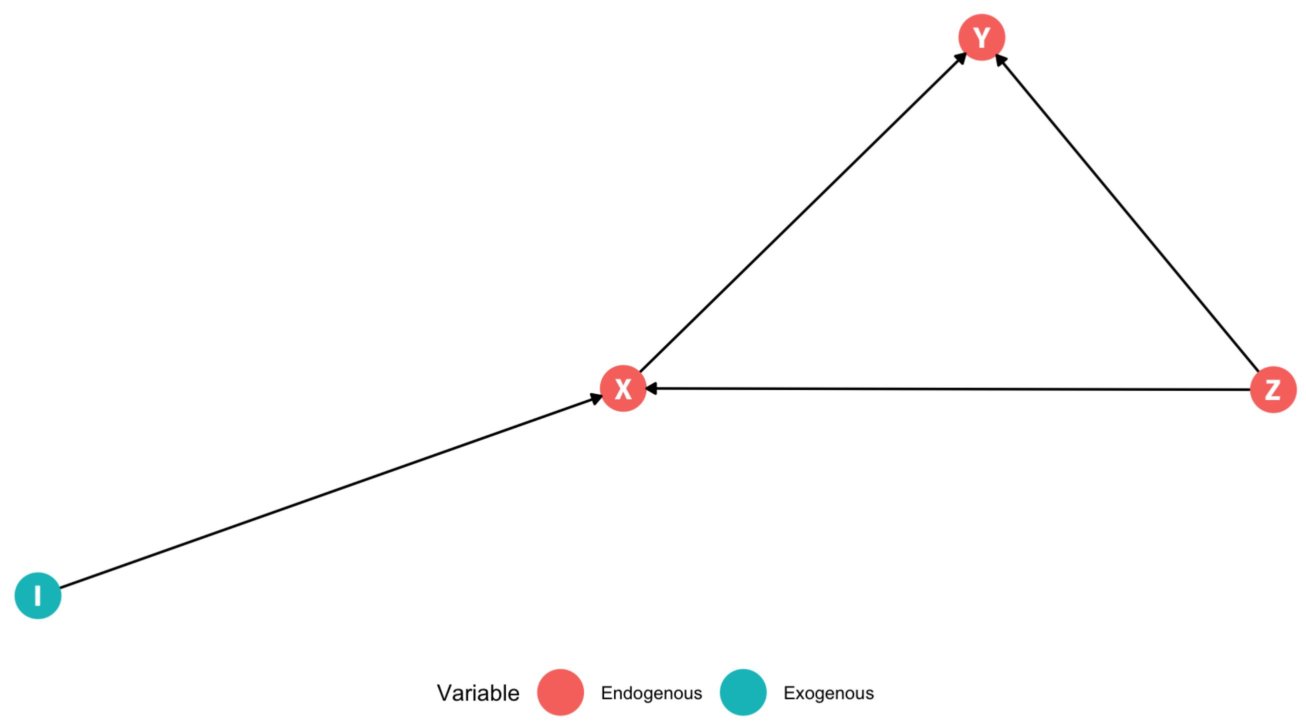
- Another strategy to is to find some source of exogenous variation that removes the endogeneity of a variable, using that source as a **instrumental variable**



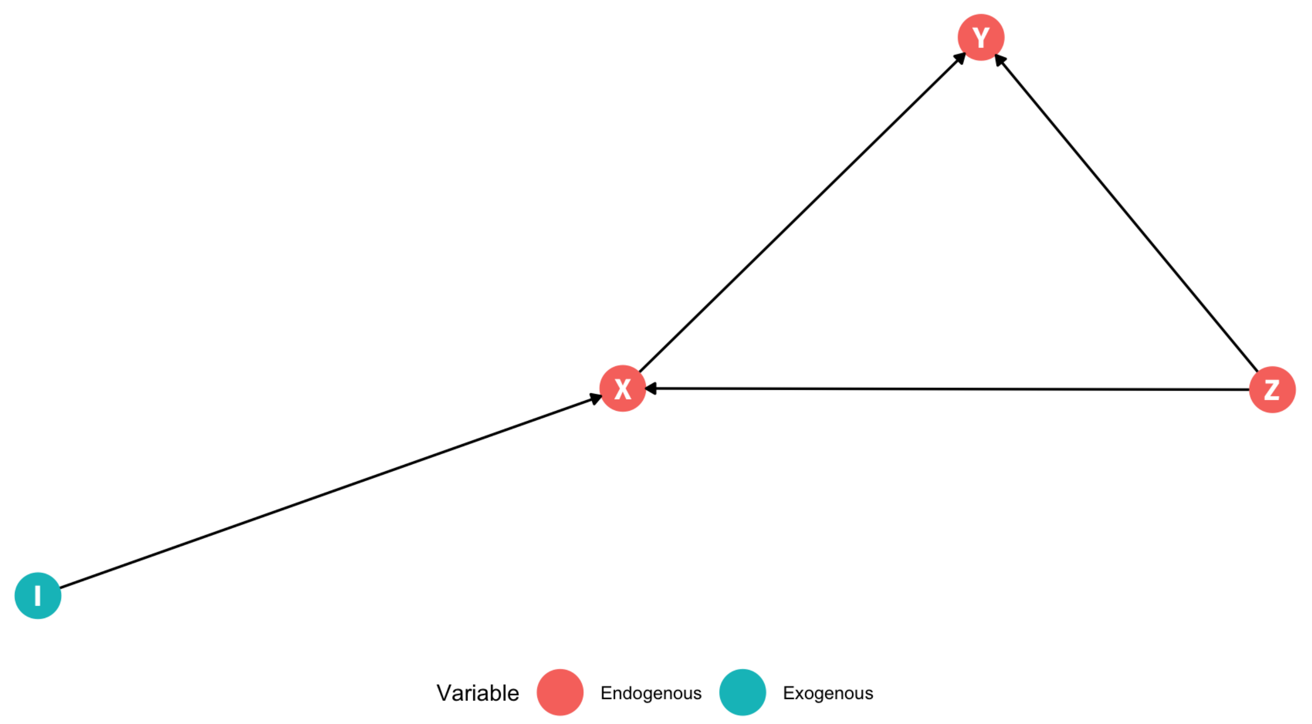
Instrumental Variables Models

Understanding Instruments



- **X** and **Y** are correlated
- Consider confounding variable **Z** that would meet the conditions of omitted variable bias:
  1. Causes Y (in error term u)
  2. Correlated with X
- Causal pathways from X to Y:
  1. $X \rightarrow Y$ (causal, front door)
  2. $X \leftarrow Z \rightarrow Y$ (non-causal, back door)
- Consider variable **I** which causes **X** but *not* **Y**

Understanding Instruments



- Variable **I** has no backdoors between it and Y
- The only way to reach Y from I is through X:
  - $I \rightarrow X \rightarrow Y$
- Variable **I** is a good **instrument** for X if it satisfies two conditions:

1. **Inclusion condition**: I statistically-significantly explains X
2. **Exclusion condition**: I is uncorrelated with u, so it does not directly affect Y
   - I only affects Y *through* its effect on X

Example I: Veterans' Earnings

Example

How does veteran status affect lifetime earnings?

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{ Veteran}_i + \text{u}_i$$

- Veteran_i is endogenous, correlated with other things in u_i
  - Choice to enlist in military for non-random reasons

Example I: Exogeneous and Endogenous Variation

- Imagine if we could split variation in Veteran_i into an exogenous part and an endogenous part:

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Veteran}_i + u_i$$

Example I: Exogeneous and Endogenous Variation

- Imagine if we could split variation in Veteran_i into an exogenous part and an endogenous part:

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Veteran}_i + u_i$$
$$\text{Earnings}_i = \beta_0 + \beta_1 (\text{Veteran}_i^{Ex.} + \text{Veteran}_i^{End.}) + u_i$$

Example I: Exogeneous and Endogenous Variation

- Imagine if we could split variation in Veteran_i into an exogenous part and an endogenous part:

$$\text{Earnings}_i = \beta_0 + \beta_1 \, \text{Veteran}_i + u_i$$
$$\text{Earnings}_i = \beta_0 + \beta_1 \, (\text{Veteran}_i^{Ex.} + \text{Veteran}_i^{End.}) + u_i$$
$$\text{Earnings}_i = \beta_0 + \beta_1 \, \text{Veteran}_i^{Ex.} + \underbrace{\beta_1 \text{Veteran}_i^{End.} + u_i}_{w_i}$$

Example I: Exogeneous and Endogenous Variation

- Imagine if we could split variation in Veteran_i into an exogenous part and an endogenous part:

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Veteran}_i + u_i$$
$$\text{Earnings}_i = \beta_0 + \beta_1 (\text{Veteran}_i^{Ex.} + \text{Veteran}_i^{End.}) + u_i$$
$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Veteran}_i^{Ex.} + \underbrace{\beta_1 \text{Veteran}_i^{End.} + u_i}_{w_i}$$

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Veteran}_i^{Ex.} + w_i$$

- What would a plausible source of $\text{Veteran}_i^{Ex.}$ be?
- Choices to enlist in the military for "random" reasons, uncorrelated with u_i (other things that affect Earnings_i)

Inclusion & Exclusion Conditions for Instruments

- We isolate the *exogenous variation* in X_i with an **instrumental variable** that is:

1. Correlated with the explanatory variable (**relevance**)
   - Often called the **"inclusion condition"**
2. Uncorrelated with the error term (**exogenous**)
   - Often called the **"exclusion condition"**

- So for our example:

Tip

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{ Veteran}_i + \text{u}_i$$

We want an instrument I for Veteran$_i$ which is:

1. **Relevant:** $cor(Veteran_i, I) \neq 0$
2. **Exogenous:** $cor(I, u_i) \neq 0$

Example Instrument: Relevance

- **Relevance ("inclusion condition")**: we need I to vary with our endogenous X variable
- We can *test* this condition using a regression and t-test on the relevant coefficient (checking correlations also helps)

Example

For Veteran_i status, consider several potential I variables:

1. Social security number

Probably not relevant
uncorrelated with military service

2. Physical fitness

Possibly relevant
may be correlated with military service

3. Vietnam War Draft

Relevant
being drawn in draft causes military service

Example Instrument: Exogeneity

- **Exogeneity ("exclusion condition")** we need I to be "as good as randomly assigned", uncorrelated with u (other factors that determine Y)
- **This is not testable!** (Need a good argument from theory/intuition)
  - Does I *only* affect Y through X?

Example

For Veteran_i status, consider several potential I variables:

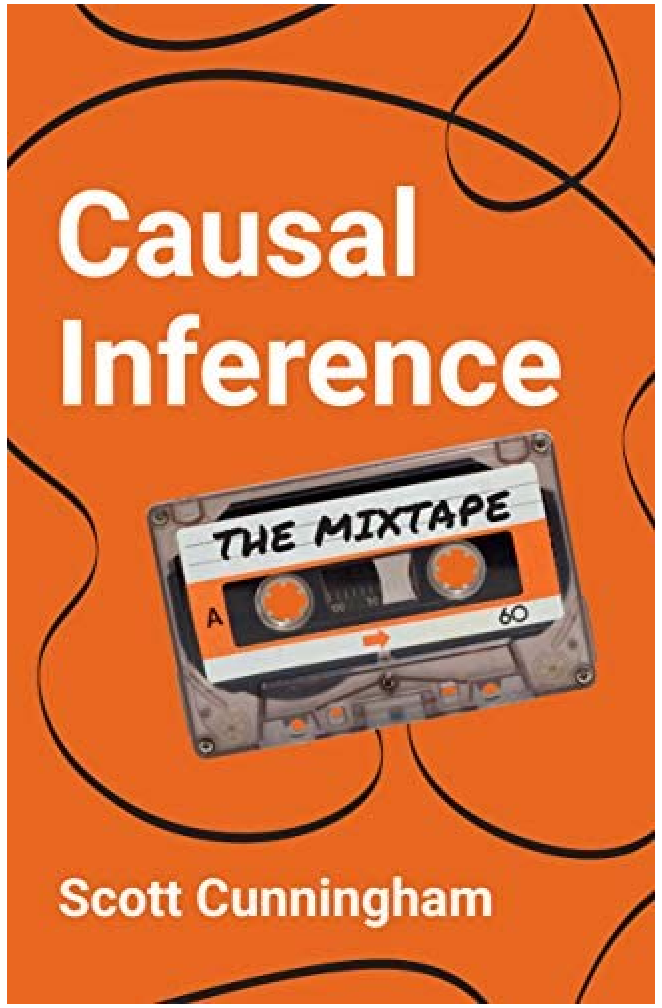
1. Social security number

Exogenous
uncorrelated with other factors of earnings

2. Physical fitness

Not exogeous
correlated with many other factors of earnings

Exogeneity: The “Huh?” Factor



> “A necessary but not a sufficient condition for having an instrument that can satisfy the exclusion restriction is if people are confused when you tell them about the instrument’s relationship to the outcome,” (p.123).

Cunningham, Scott, 2021, *Causal Inference: The Mixtape*

Good Instruments are Hard to Find (And Weird) I

Outcome	Endogenous Variable	Unobservables	Instrument
Income	Education	Ability	Quarter of birth
Income	Education	Ability	Father’s education
Income	Education	Ability	Distance to college
Income	Education	Ability	Military draft
Health	Smoking	Other negative health behaviors	Tobacco taxes
Crime rates	Patrol hours	number of criminals	Election cycles
Crime rates	Patrol hours	number of criminals	Firefighters
Crime rates	Patrol hours	number of criminals	Terror Alert levels
Crime rates	Incarceration rates	Simultaneous causality	Overcrowding litigations
Labor market success	Americanization	Ability	Scrabble score of name
Conflict	Economic growth	Simultaneous causality	Rainfall

Good Instruments are Hard to Find (And Weird) II

Table 1
Examples of Studies That Use Instrumental Variables to Analyze Data From Natural and Randomized Experiments

Outcome Variable	*Endogenous Variable*	*Source of Instrumental Variable(s)*	*Reference*
		1. Natural Experiments	
Labor supply	Disability insurance replacement rates	Region and time variation in benefit rules	Gruber (2000)
Labor supply	Fertility	Sibling-Sex composition	Angrist and Evans (1998)
Education, Labor supply	Out-of-wedlock fertility	Occurrence of twin births	Bronars and Grogger (1994)
Wages	Unemployment insurance tax rate	State laws	Anderson and Meyer (2000)
Earnings	Years of schooling	Region and time variation in school construction	Duflo (2001)
Earnings	Years of schooling	Proximity to college	Card (1995)
Earnings	Years of schooling	Quarter of birth	Angrist and Krueger (1991)
Earnings	Veteran status	Cohort dummies	Imbens and van der Klaauw (1995)
Earnings	Veteran status	Draft lottery number	Angrist (1990)
Achievement test scores	Class size	Discontinuities in class size due to maximum class-size rule	Angrist and Lavy (1999)
College enrollment	Financial aid	Discontinuities in financial aid formula	van der Klaauw (1996)
Health	Heart attack surgery	Proximity to cardiac care centers	McClellan, McNeil and Newhouse (1994)
Crime	Police	Electoral cycles	Levitt (1997)
Employment and Earnings	Length of prison sentence	Randomly assigned federal judges	Kling (1999)
Birth weight	Maternal smoking	State cigarette taxes	Evans and Ringel (1999)

Angrist, Joshua D and Alan B Kreuger, 2001, "Instrumental Variables and the Search for Identification: From Supply and Demand to Natural Experiments," *Journal of Economic Perspectives* 15(4): 69-85

Exogeneity: The “Huh?” Factor

Good Instruments are Hard to Find (And Weird) III

Rain, Rain, Go away: 137 potential exclusion-restriction violations for studies using weather as an instrumental variable

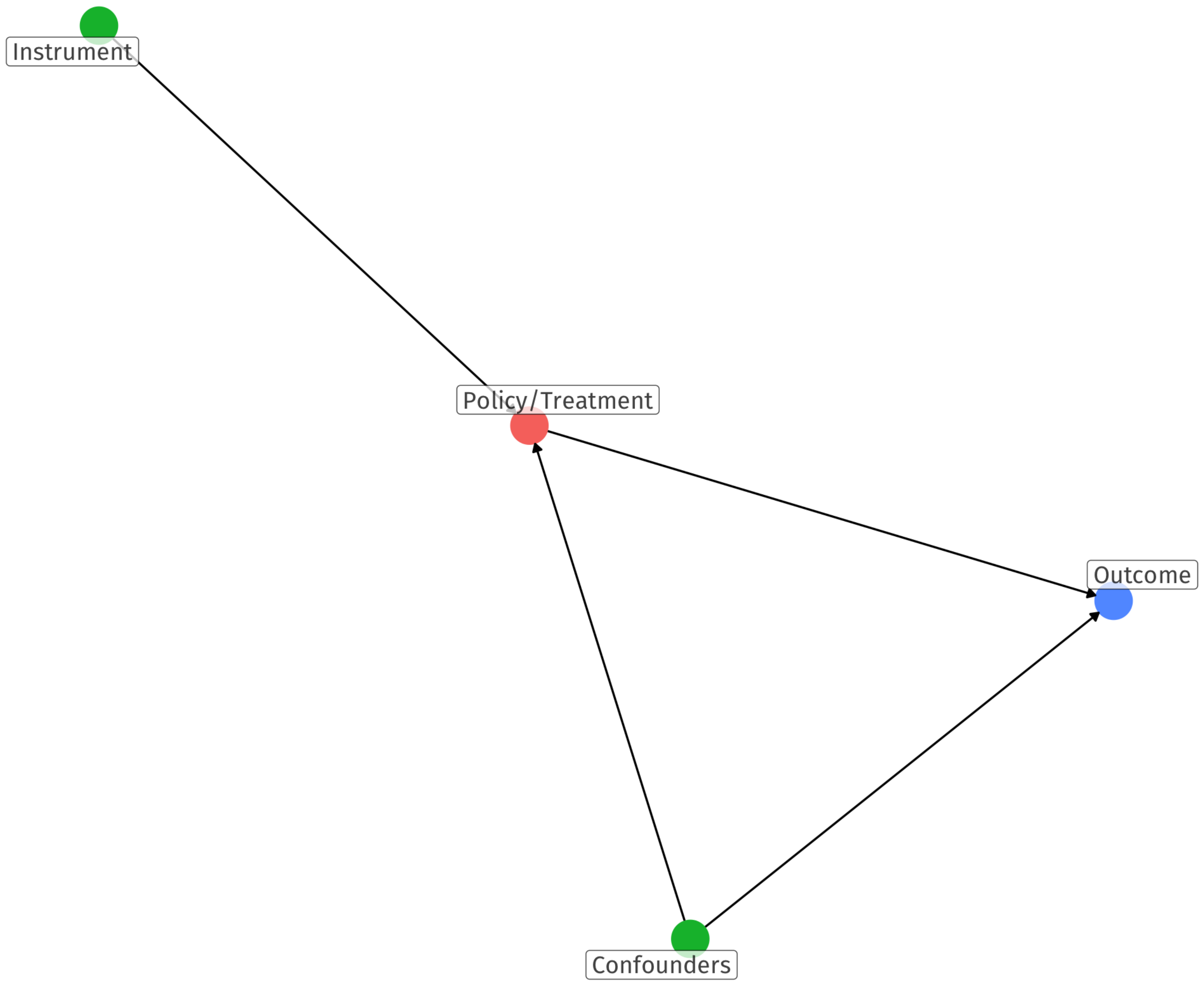
Jonathan Mellon (University of Manchester)

20-10-2020

Abstract

Instrumental variable (IV) analysis assumes that the instrument only affects the dependent variable via its relationship with the independent variable. Other possible causal routes from the IV to the dependent variable are exclusion-restriction violations and make the instrument invalid. Weather has been widely used as an instrumental variable in social science to predict many different variables. The use of weather to instrument different independent variables represents strong prima facie evidence of exclusion violations for all studies using weather as an IV. A review of 185 social science studies reveals 137 variables which have been linked to weather, all of which represent potential exclusion violations. I conclude with practical steps for systematically reviewing existing literature to identify possible exclusion violations when using IV designs.

"Testing" the Exclusion Restriction



- Can you argue that the instrument does **not** affect outcome Y **except only** through X?

Examples

- Instrument → ? → outcome
- Quarter of birth → ? → wages
- Rainfall → ? → civil war
- Scrabble score of name → ? → wages

Example: Review

- Instrument must be
  1. Correlated with our endogenous variable (X_i) (inclusion restriction)
  2. Uncorrelated with omitted variables that affect Y_i (exclusion restriction)
- To summarize: **the instrument only affects the outcome through its relationship with the endogenous variable**

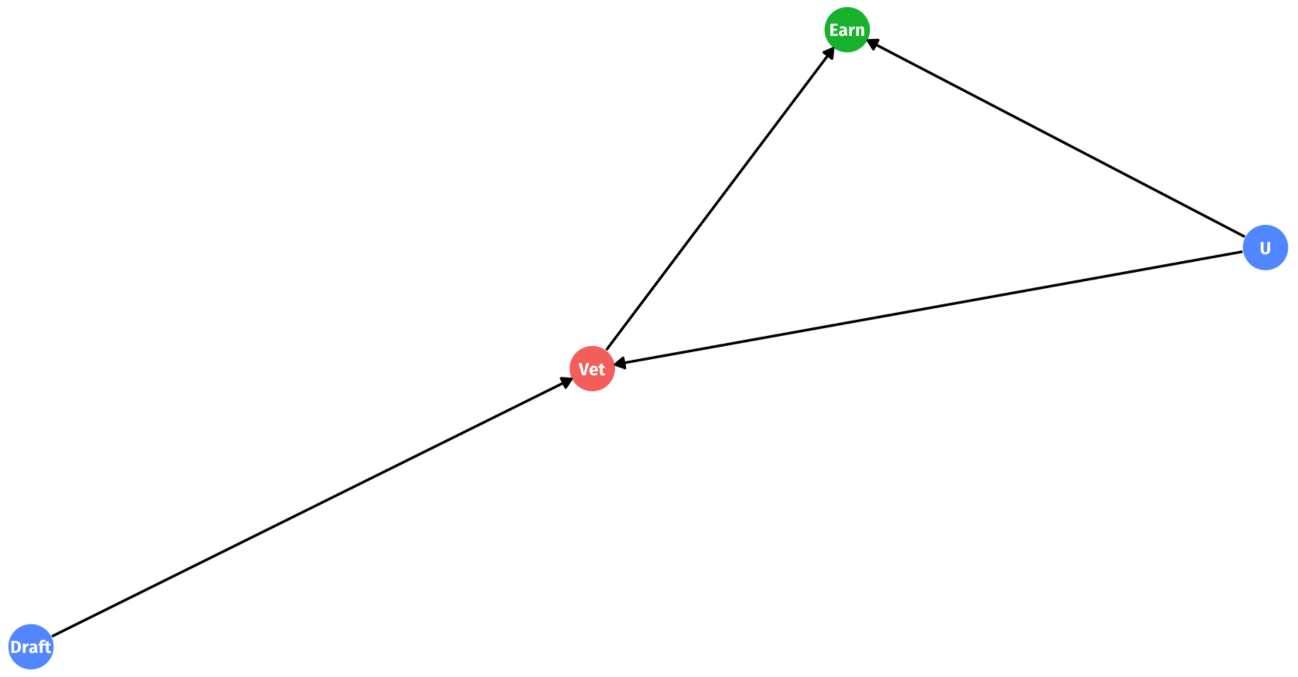
Example

For Veteran_i status, our several potential I variables:

1. Social security number: **Not relevant** **Exogenous**
2. Physical fitness: **Relevant** **Not exogenous**
3. Vietnam War Draft: **Relevant** **Exogenous**

- The Vietnam War Draft is the only ***valid* instrument**

Example I: DAG Form



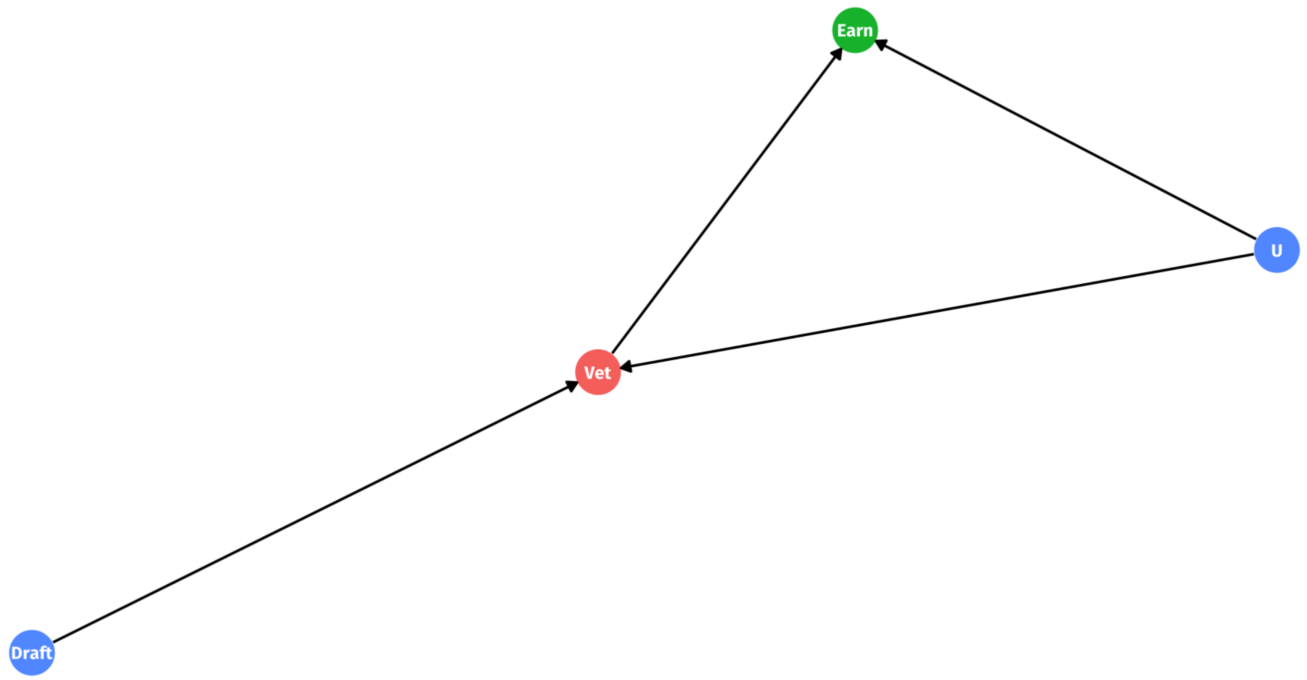
- Causal pathways from X to Y:
  1. $Vet \rightarrow Earn$
  2. $Vet \leftarrow U \rightarrow Earn$
- We want the causal effect of

$$Vet \rightarrow Earn$$

- With our instrument

$$Draft \rightarrow Vet \rightarrow Earn$$

Example I: DAG Form



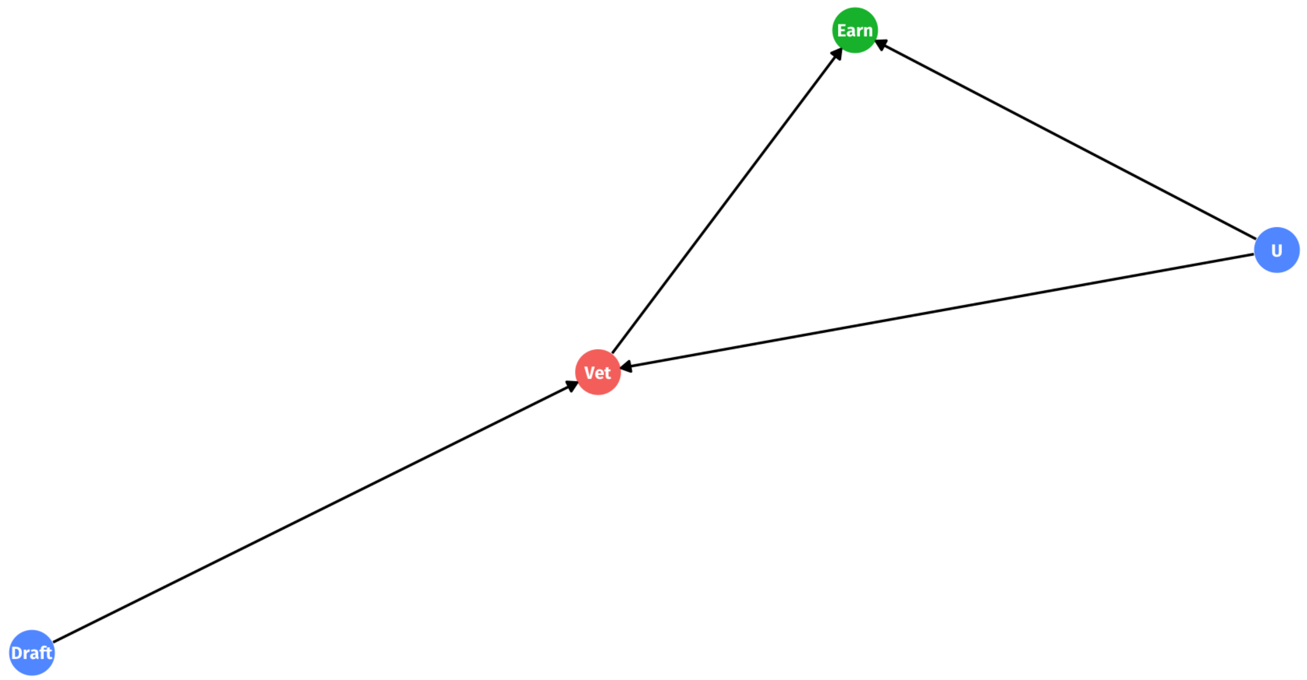
- With our instrument

$$Draft \rightarrow Vet \rightarrow Earn$$

- Based on our assumptions on independence and exogeneity:

(Effect of draft on earnings) =
(Effect of draft on veteran) × (Effect of veteran on earnings)

Example I: DAG Form



- With our instrument

$$Draft \rightarrow Vet \rightarrow Earn$$

- Based on our assumptions on independence and exogeneity:

$$\text{(Effect of draft on earnings)} = \text{(Effect of draft on veteran)} \times \text{(Effect of veteran on earnings)}$$

- To find effect of veteran on earnings, rearrange!

$$\text{(Effect of veteran on earnings)} = \frac{\text{(Effect of draft on earnings)}}{\text{(Effect of draft on veteran)}}$$

Estimating The Effect With Instrumental Variables

Recall: We want to estimate the effect of veteran status on earnings.

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{ Veteran}_i + u_i$$

- Consider two other relationships:

1. Effect of instrument on the endogenous variable

$$\text{Veteran}_i = \gamma_0 + \gamma_1 \text{ Draft}_i + w_i$$

2. Effect of instrument on the outcome variable (**"reduced form"**)

$$\text{Earnings}_i = \pi_0 + \pi_1 \text{Draft}_i + v_i$$

- Using these, we can estimate our desired effect, (Effect of veteran status on earnings):

$$\beta_1^{IV} = \frac{\pi_1}{\gamma_1}$$

Estimating The Effect With Instrumental Variables

- With our instrument, we estimate β_1 using

$$\hat{\beta}_1^{IV} = \frac{\hat{\pi}_1}{\hat{\gamma}_1}$$

where $\hat{\pi}_1$ and $\hat{\gamma}_1$ come from the regressions in the last slide

- Is this estimator unbiased?

$$\mathbb{E}[\hat{\beta}_1^{IV}] = \beta_1 + \frac{\text{cov}(Instrument, u)}{\text{cov}(\text{Instrument}, \text{Endog. variable})}$$

- Yes: so long as the instrument is **valid**, i.e. **exogenous** (numerator) and **relevant** (denominator)

Example: Education

Example

Consider the age-old question of how education affects wages.

$$\text{wage}_i = \beta_0 + \beta_1 \text{ education}_i + u_i$$

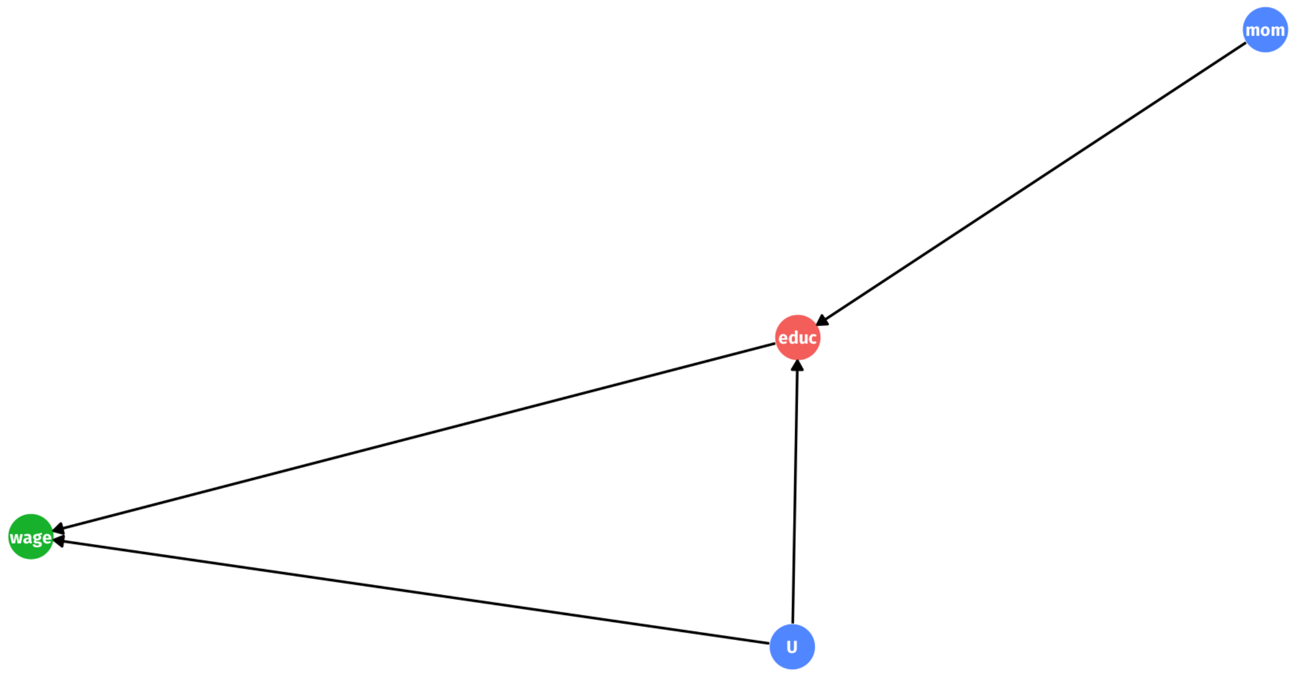
```
ols_reg <- lm(wage ~ education, data = wage_df)
tidy(ols_reg)
```

term <chr>	estimate <dbl>	std.error <dbl>
(Intercept)	176.50395	89.151950
education	58.59393	6.439262

2 rows | 1-3 of 5 columns

- `education` is endogenous

Example: Instrument



- Causal pathways from $educ$ to $wage$:
  1. $educ \rightarrow wage$
  2. $educ \leftarrow U \rightarrow wage$
- We want the causal effect of

$$educ \rightarrow wage$$

- With our instrument

$$mom \rightarrow educ \rightarrow wage$$

Example: Relevance

- We can check the **relevance** of mother's education as an instrument for education
- This regression is known as the **"first stage"**: effect of the instrument on the endogenous variable

$$\text{Education}_i = \gamma_0 + \gamma_1 \text{Mother's education}_i + v_i$$

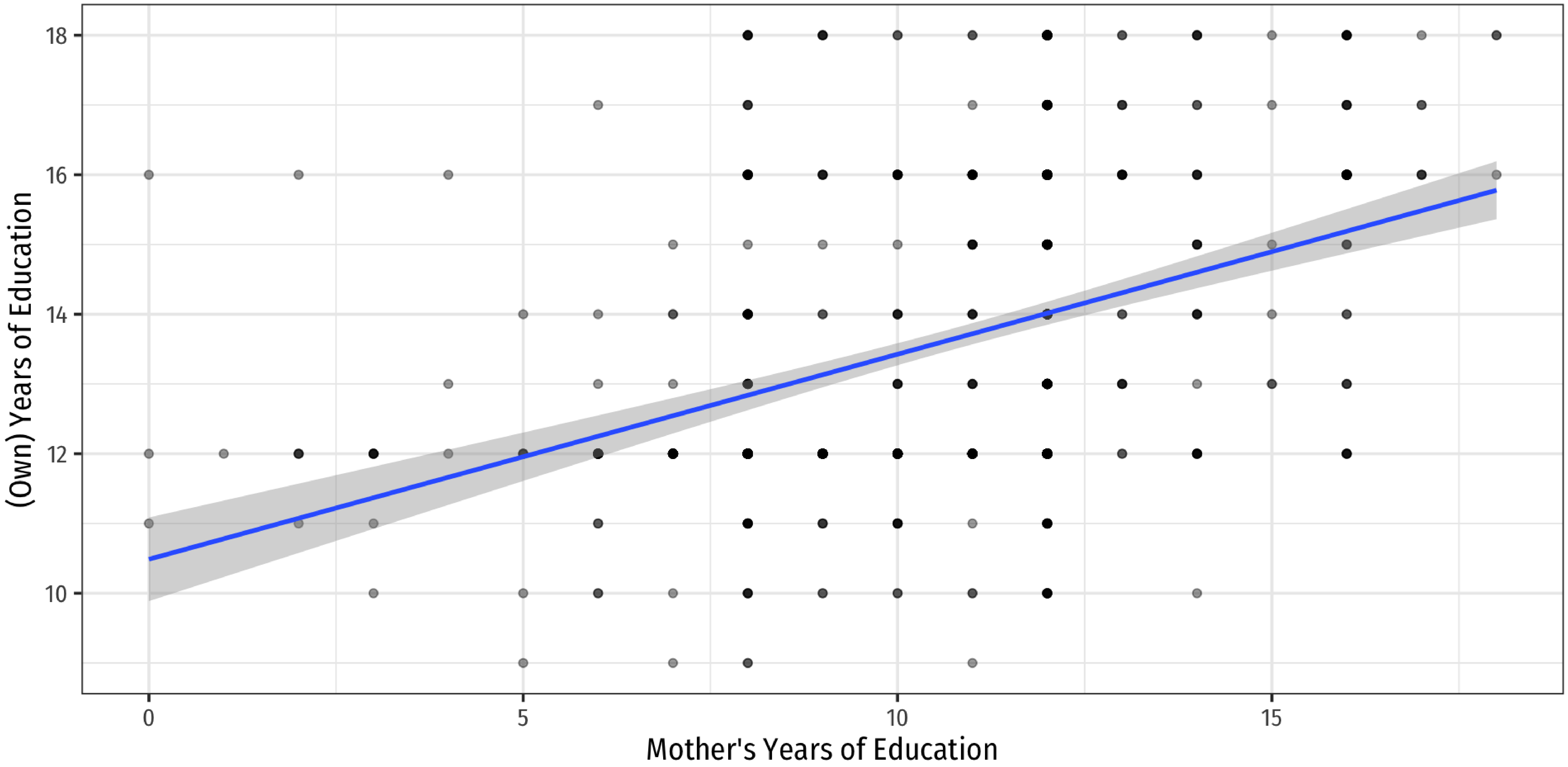
```r
first_stage <- lm(education ~ education_mom, data = wage_df)
tidy(first_stage)
```

term <chr>	estimate <dbl>
(Intercept)	10.4867466
education_mom	0.2939719

2 rows | 1-2 of 5 columns

- p-value suggests this is a very relevant instrument!

First-Stage Visualized



Exogeneity

- We need our instrument, mother's education to be **exogenous**

1. Mother's education must only affect wages through (own) education
2. Mother's education must be uncorrelated with other factors that affect wages (i.e. the error term u_i)

- We want to be able to compare two individuals A and B whose mothers have different levels of education and say their *only differences* between A and B are their mothers' education levels.

Reduced Form

- The estimate for the **reduced form** (effect of instrument on outcome)

$$\text{Wage}_i = \pi_0 + \pi_1 \text{Mother's education}_i + v_i$$

term <chr>	estimate <dbl>
(Intercept)	633.33672
education_mom	31.81183

2 rows | 1-2 of 5 columns

The Effect We're After

- So what's our estimate of the returns to education on wages

$$\text{Wages}_i = \beta_0 + \beta_1 \, \text{Education}_i + u_i$$

- We know the IV estimate for β_1 is

$$\beta_1^{IV} = \frac{\pi_1}{\gamma_1}$$

1. In the reduced form equation, we estimated $\hat{\pi}_1 \approx 31.81$
2. In the first-stage equation, we estimated $\hat{\gamma}_1 \approx 0.294$

$$\hat{\beta}_1^{IV} = \frac{\hat{\pi}_1}{\hat{\gamma}_1} \approx \frac{31.81}{0.294} \approx 108.2$$

Example in R: `estimatr`

- `estimatr` package

```r
library(estimatr)

iv_reg <- iv_robust(wage ~ education | education_mom, data = wage_df)
summary(iv_reg)
```

```
Call:
iv_robust(formula = wage ~ education | education_mom, data = wage_df)

Standard error type:  HC2

Coefficients:
            Estimate Std. Error t value  Pr(>|t|) CI Lower CI Upper  DF
(Intercept)   -501.5     226.48  -2.214 2.712e-02  -946.11   -56.84 720
education      108.2      16.81   6.437 2.220e-10    75.21   141.22 720

Multiple R-squared:  0.02917 ,  Adjusted R-squared:  0.02783 
F-statistic: 41.44 on 1 and 720 DF,  p-value: 2.22e-10
```

```r
tidy(iv_reg)
```

term <chr>	estimate <dbl>	std.error <dbl>	statistic <dbl>
(Intercept)	-501.4743	226.47608	-2.214248
education	108.2138	16.81031	6.437348

2 rows | 1-4 of 9 columns

Example in R: `fixest`

- `fixest` package

```r
library(fixest)

iv_reg_2 <- feols(wage ~ 1 | education ~ education_mom, data = wage_df)
summary(iv_reg_2)
```

```
TSLS estimation, Dep. Var.: wage, Endo.: education, Instr.: education_mom
Second stage: Dep. Var.: wage
Observations: 722
Standard-errors: IID
              Estimate Std. Error  t value   Pr(>|t|)
(Intercept)   -501.474   246.6842 -2.03286 4.2433e-02 *
fit_education  108.214    18.0210  6.00486 3.0367e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 401.8   Adj. R2: 0.027825
F-test (1st stage), education: stat = 115.5    , p < 2.2e-16 , on 1 and 720 DoF.
                  Wu-Hausman: stat =   9.63706, p = 0.001982, on 1 and 719 DoF.
```

```r
tidy(iv_reg_2)
```

term <chr>	estimate <dbl>	std.error <dbl>	statistic <dbl>
(Intercept)	-501.4743	246.68425	-2.032859
fit_education	108.2138	18.02104	6.004860

2 rows | 1-4 of 5 columns

Two-Stage Least Squares (2SLS)

Instrumental Variables & 2SLS

- Now we know how to use instruments (when there is **1** endogenous X variable, and **1** instrumental variable I):

  1. Estimate reduced form (regress outcome $\sim$ instrument)
  2. Estimate first stage (regress endog. variable $\sim$ instrument)
  3. Calculate IV-estimate of outcome $\sim$ endog. variable using (1) and (2)

- Instrument isolates only the exogenous variation in the endogenous variable

- What if we want to use **multiple** endogenous variables and/or **multiple** instruments?

- Extend this approach using **two-stage least squares (2SLS)**[1]

1. In practice, since 2SLS is used so commonly, most people conflate instrumental variables approaches with 2SLS, but it is just *one* approach to using instruments

Intuitions from Instruments & 2SLS

- We already have a lot of intuitions from IV to talk about 2SLS:

$$\begin{aligned}
\text{Endogenous model} \qquad \text{Outcome}_i &= \beta_0 + \beta_1 (\text{Endog. var.})_i + u_i \\
\text{First stage} \qquad (\text{Endog. var.})_i &= \pi_0 + \pi_1 \text{Instrument}_i + v_i \\
\text{Second stage} \qquad \text{Outcome}_i &= \delta_0 + \delta_1 \widehat{(\text{Endog. var.})}_i + \varepsilon_i \\
\text{Reduced form} \qquad \text{Outcome}_i &= \pi_0 + \pi_1 \text{Instrument}_i + w_i
\end{aligned}$$

where $\widehat{(\text{Endog. var.})}_i$ are the predicted values (*fitted values*) from the first-stage regression

2SLS: Advantages

- 2SLS is very flexible:
  - Can add additional endogenous variables
  - Can use additional instruments for endogenous variables
  - Can add additional (exogenous) control variables $(X_2, \cdots, X_k)$
- Of course, your instruments still need to be **valid**:

1. Exogenous
2. Relevant

2SLS: Multiple Instruments

Example

Come back to to our returns to education on wages example.

$$\text{wage}_i = \beta_0 + \beta_1 \text{ education}_i + u_i$$

- Suppose both mother’s education and father’s education are **valid** instruments (relevant and exogenous)
- Then the **first stage** regression is:

$$\text{Education}_i = \gamma_0 + \gamma_1 \text{Mother's education}_i + \gamma_2 \text{Father's education}_i + v_i$$

term <chr>	estimate <dbl>	std.error <dbl>	statistic <dbl>
(Intercept)	9.8453600	0.30470880	32.310718
education_mom	0.1486908	0.03215931	4.623569
education_dad	0.2156354	0.02751775	7.836229

3 rows | 1-4 of 5 columns

First Stage: Checking Relevance

term <chr>	estimate <dbl>	std.error <dbl>	statistic <dbl>
(Intercept)	9.8453600	0.30470880	32.310718
education_mom	0.1486908	0.03215931	4.623569
education_dad	0.2156354	0.02751775	7.836229

3 rows | 1-4 of 5 columns

- Both instruments appear to be relevant (small p-values), but we can more formally test their relevance **jointly** (i.e., an F-test)

```r
library(car)
linearHypothesis(first_stage_multiple_IVs, c("education_mom", "education_dad"))
```

	Res.Df <dbl>	RSS <dbl>	Df <dbl>	Sum of Sq <dbl>	F <dbl>
1	721	3607.215	NA	NA	NA
2	719	2864.067	2	743.1482	93.28057

2 rows | 1-6 of 7 columns

- p-value is small, so they are jointly significant, i.e. relevant instruments

Aside: The Problem of Weak Instruments

- **Weak instruments** have low relevance (i.e. $\text{cor}(X, I)$ is weak) and add little explanatory power
- This can make OLS (and 2SLS) unreliable in small samples, and significantly raises the variance of OLS estimates
- This likelihood also increases when we have multiple instruments, or more instruments than endogenous variables (a problem of **"overidentification"**)

Second-Stage

```
# add fitted values from first stage
wage_df$education_hat <- first_stage_multiple_IVs$fitted.values
```

- Now run the **second stage** regression:

$$\text{Wage}_i = \delta_0 + \delta_1 \widehat{(\text{education})}_i + \varepsilon_i$$

term <chr>	**estimate** <dbl>
(Intercept)	-454.6828
education_hat	104.7893

2 rows | 1-2 of 5 columns

Comparing Results

	OLS	IV	2SLS (two instruments)
Constant	176.50**	−501.47**	−454.68**
	(89.15)	(226.48)	(198.15)
education	58.59***	108.21***	104.79***
	(6.44)	(16.81)	(14.46)
n	722	722	722
Adj. R^2	0.10	0.03	0.07
SER	386.21	401.82	393.71

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Using `estimatr` or `fixest`

- You can do this "by hand" as we did, but R packages will run both stages for you
- `estimatr` package: `iv_robust(y ~ x1 + x2 + ... | z1 + z2 + ..., data = df)`
  - `x1`, `x2`, `...` are your endogenous variables
  - `z1`, `z2`, `...` are instruments
  - `df` is the dataframe

```r
iv_robust(wage ~ education | education_mom + education_dad, data = wage_df)
```

```
             Estimate Std. Error   t value     Pr(>|t|)   CI Lower  CI Upper
(Intercept) -454.6828  199.94577 -2.274030 2.325766e-02 -847.22915 -62.13638
education    104.7893   14.85244  7.055357 4.051281e-12   75.62999 133.94851
             DF
(Intercept) 720
education   720
```

Using estimatr or fixest

- fixest package: feols()

```
feols(wage ~ 1 | education ~ education_mom + education_dad, data = wage_df)
```

```
TSLS estimation, Dep. Var.: wage, Endo.: education, Instr.: education_mom, education_dad
Second stage: Dep. Var.: wage
Observations: 722
Standard-errors: IID
              Estimate Std. Error  t value   Pr(>|t|)
(Intercept)   -454.683   201.2012 -2.25984 2.4129e-02 *
fit_education  104.789    14.6851  7.13576 2.3529e-12 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 399.8   Adj. R2: 0.037696
F-test (1st stage), education: stat = 93.3     , p < 2.2e-16 , on 2 and 719 DoF.
                   Wu-Hausman: stat = 13.6     , p = 2.448e-4, on 1 and 719 DoF.
                       Sargan: stat =  0.111147, p = 0.738842, on 1 DoF.
```

Another Example: Levitt (2002) I

Example

How do police affect crime?

$$Crime_{it} = \beta_0 + \beta_1 Police_{it} + u_{it}$$

- Police→crime (more police reduces crime)
- Crime→Police (high crime areas tend to have more police)
- $cor(Police, \epsilon) \neq 0$: population, income per capita, drug use, recessions, demography, etc.

Another Example: Levitt (2002) II

- Levitt (2002): use number of firefighters as an instrumental variable
- Some police are hired for **endogenous** reasons (respond to crime, changes in economy, demographics, etc)
- Some police are hired for **exogenous** reasons (city just gains a larger budget and so hires more police)
  - These exogenous dynamics affect the number of firefighters in a city — *not* due to crime, but due to excess budgets, etc.
- Isolate that portion of variation in Police that covaries with Firefighters for those **exogenous** changes (i.e. for reasons *other* than crime or its causes), see how *these* changes in Police affect crime

Levitt, Steven D, (2002), "Using Electoral Cycles in Police Hiring to Estimate the Effect of Police on Crime: Reply," *American Economic Review* 92(4): 1244-1250

Another Example: Levitt (2002) III

- Levitt's (2002) paper, First Stage:

$$ln(Police_{ct}) = \gamma_1 ln(Firefighters_{ct}) + \alpha_c + \tau_t + \gamma_2 Controls_{ct} + \nu_{ct}$$

subscripts for city c at year t, two-way fixed effects: α_c city fixed-effects, τ_t year fixed-effects

- Second stage:

$$ln(\widehat{Crime}_{ct}) = \beta_1 ln(\widehat{Police}_{ct-1}) + \alpha_c + \tau_t + \beta_2 Controls_{ct} + \epsilon_{ct}$$

lag for police (last year's police force determines this year's crime rates)

Another Example: Levitt (2002) IV

TABLE 2—THE RELATIONSHIP BETWEEN FIREFIGHTE

	First-stage estimates (dependent variable = ln(Police per capita)		
Variable	(i)	(ii)	(iii)
ln(Firefighters per capita)	0.251	0.236	0.206
	(0.050)	(0.054)	(0.050)
ln(Street and highway workers per capita)	—	—	0.014
			(0.014)
ln(State prisoners per capita)	—	−0.101	−0.077
		(0.022)	(0.022)
Unemployment rate	—	0.571	0.265
		(0.276)	(0.314)
State income per capita (×10,000)	—	0.150	0.211
		(0.004)	(0.005)
Effective abortion rate	—	0.033	0.045
(×100)	(0.013)	(0.013)	(0.026)
ln(City population)	—	0.040	−0.014
		(0.040)	(0.047)
Percentage black	—	0.361	0.493
		(0.204)	(0.264)
City-fixed effects and year dummies included?	yes	yes	yes
R^2:	0.947	0.952	0.962
Number of observations:	2,032	2,032	1,445

Instrument is statistically significant ($t \approx 5$), inclusion condition met

A 1% increase in firefighters is associated with a 0.206-0.251% increase in police

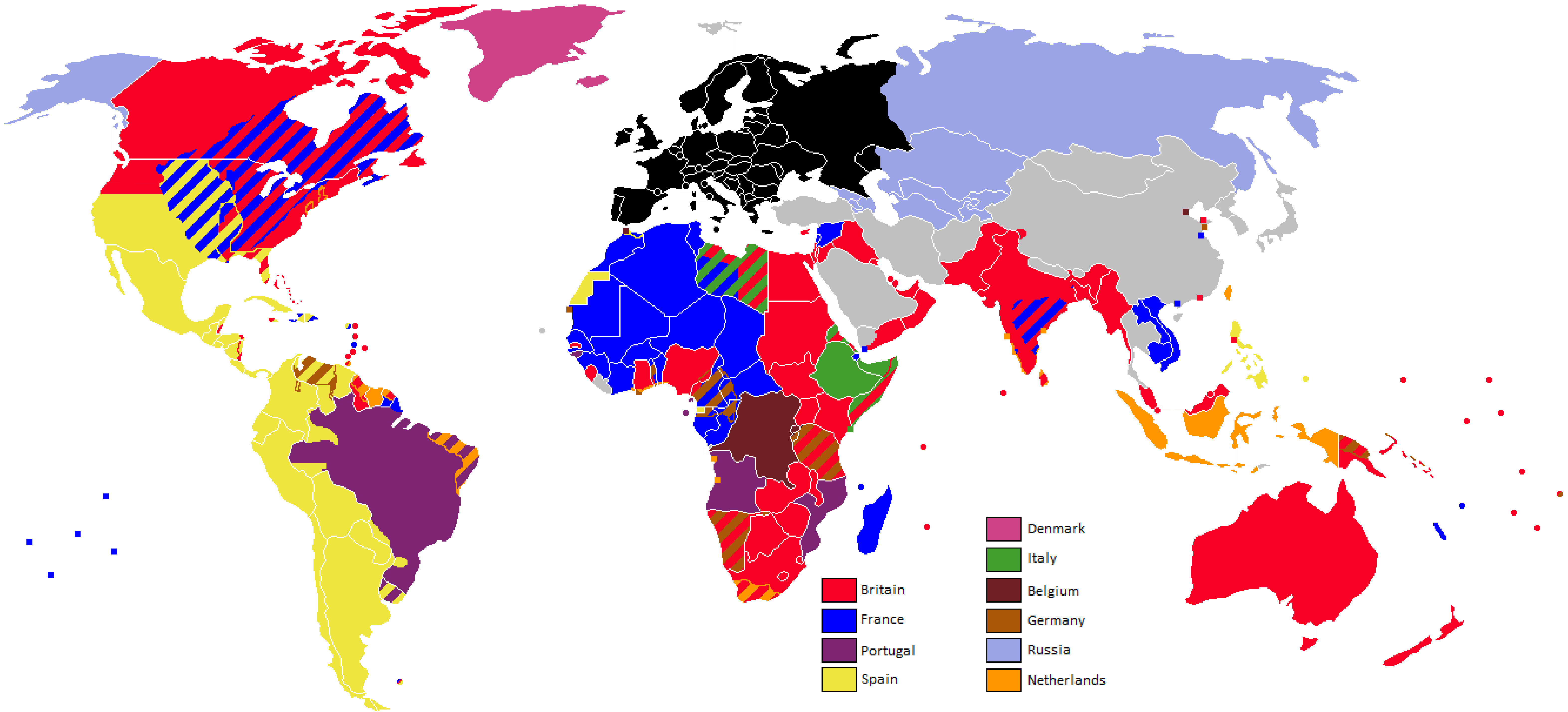
Another Example: Levitt (2002) V

TABLE 3—THE IMPACT OF POLICE ON CRIME

	Violent crime			Property crime		
Variable	OLS	OLS	IV	OLS	OLS	IV
ln(Police per capita)$_{t-1}$	0.562	−0.076	−0.435	0.113	−0.218	−0.501
	(0.056)	(0.061)	(0.231)	(0.038)	(0.052)	(0.235)
ln(State prisoners per capita)$_{t-1}$	0.250	−0.131	−0.171	0.189	−0.273	−0.305
	(0.039)	(0.036)	(0.044)	(0.030)	(0.028)	(0.037)
Unemployment rate	3.573	−0.741	−0.480	1.283	1.023	1.231
	(0.473)	(0.365)	(0.404)	(0.312)	(0.274)	(0.326)
State income per capita (×10,000)	0.050	−0.003	0.003	0.010	0.005	0.009
	(0.005)	(0.006)	(0.007)	(0.003)	(0.004)	(0.006)
Effective abortion rate (×100)	−0.214	−0.150	−0.141	−0.184	−0.118	−0.111
	(0.045)	(0.023)	(0.025)	(0.020)	(0.021)	(0.024)
ln(City population)	0.072	0.203	0.178	−0.064	−0.333	−0.355
	(0.012)	(0.063)	(0.067)	(0.006)	(0.063)	(0.066)
Percentage black	0.627	0.233	0.398	−0.136	0.411	0.517
	(0.074)	(0.334)	(0.345)	(0.057)	(0.271)	(0.291)
City-fixed effects and year dummies included?	only year dummies	yes	yes	only year dummies	yes	yes
R^2:	0.601	0.930	—	0.238	0.819	—
Number of observations:	2,005	2,005	2,005	2,032	2,032	2,032

A 1% increase in police (last year) leads to a 0.435% decrease in violent crimes, 0.501% decrease in property crimes

Another Example: AJR (2001) I



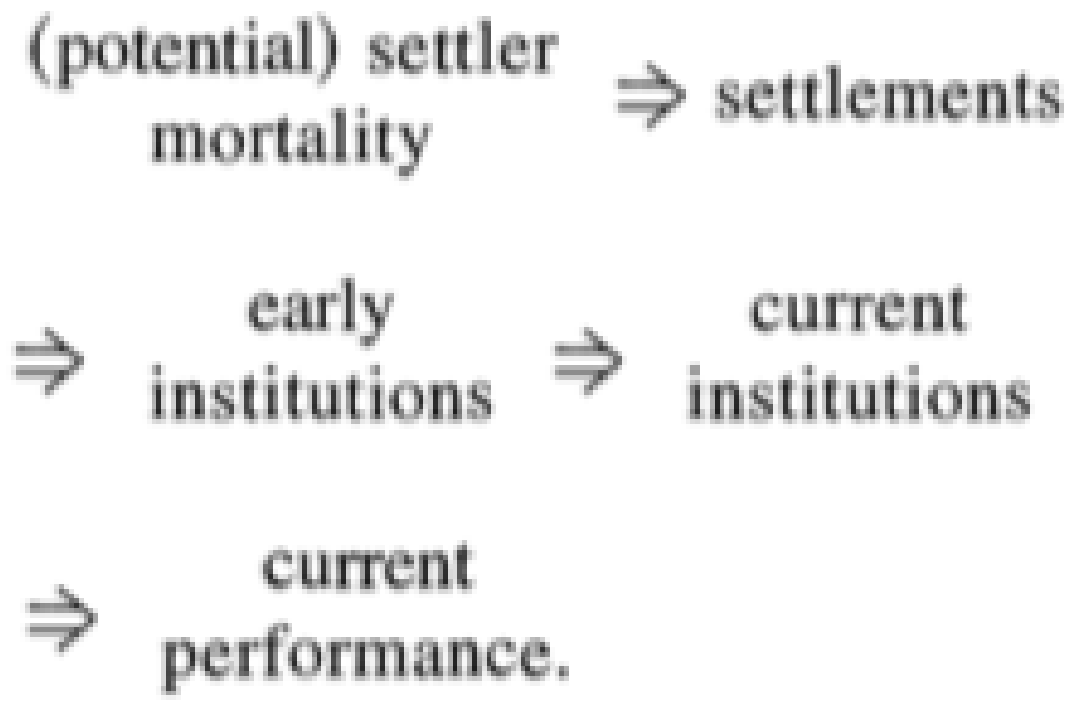
Another Example: AJR (2001) II

- Acemoglu, Johnson, and Robinson (2001): countries' wealth or poverty today depends strongly on how they were colonized.
- Europeans set up one of two types of colonies depending on the disease environment of the country:
  - **"Extractive institutions"**: Europeans facing high mortality rates set up extractive colonies primarily to exploit indigenous population to mine resources to ship back to Europe
  - **"Inclusive institutions"**: Europeans facing low mortality rates set up inclusive colonies primarily for settlement and promoting open access to trade and politics
- Those initial colonies carried through to institutions in present countries; inclusive colonies grew wealthy, extractive colonies remain stagnant

Acemoglu, Daron, Simon Johnson, and James A Robinson, (2001), "The Colonial Origins of Comparative Development: An Empirical Investigation," *American Economic Review* 91(5): 1369-1401

Another Example: AJR (2001) III

- Instrument: Settler Mortality in 1500
- **Inclusion Restriction**: Settler mortality in 1500 determines risk of expropriation today
- {**Exclusion Restriction**: Settler mortality in 1500 **does not** affect Present GDP
  - Settler mortality in 1500 **only** affects Present GDP **through** institutions determined by historical path set by settler mortality rates



Acemoglu, Daron, Simon Johnson, and James A Robinson, (2001), "The Colonial Origins of Comparative Development: An Empirical Investigation," *American Economic Review* 91(5): 1369-1401

Another Example: AJR (2001) IV

- First Stage:

$$\text{Expropriation Risk}_i = \gamma_0 + \gamma_1 ln(\text{Settler Mortality in 1500}_i) + \gamma_2 Controls + \nu_i$$

- Second Stage:

$$\ln(\text{Present GDP per capita}) = \beta_0 + \beta_1 \widehat{\text{Expropriation Risk}_i} + \cdots + \beta_2 \text{Controls} + u_i$$

Another Example: AJR (2001) V

Relationship Between Y and IV

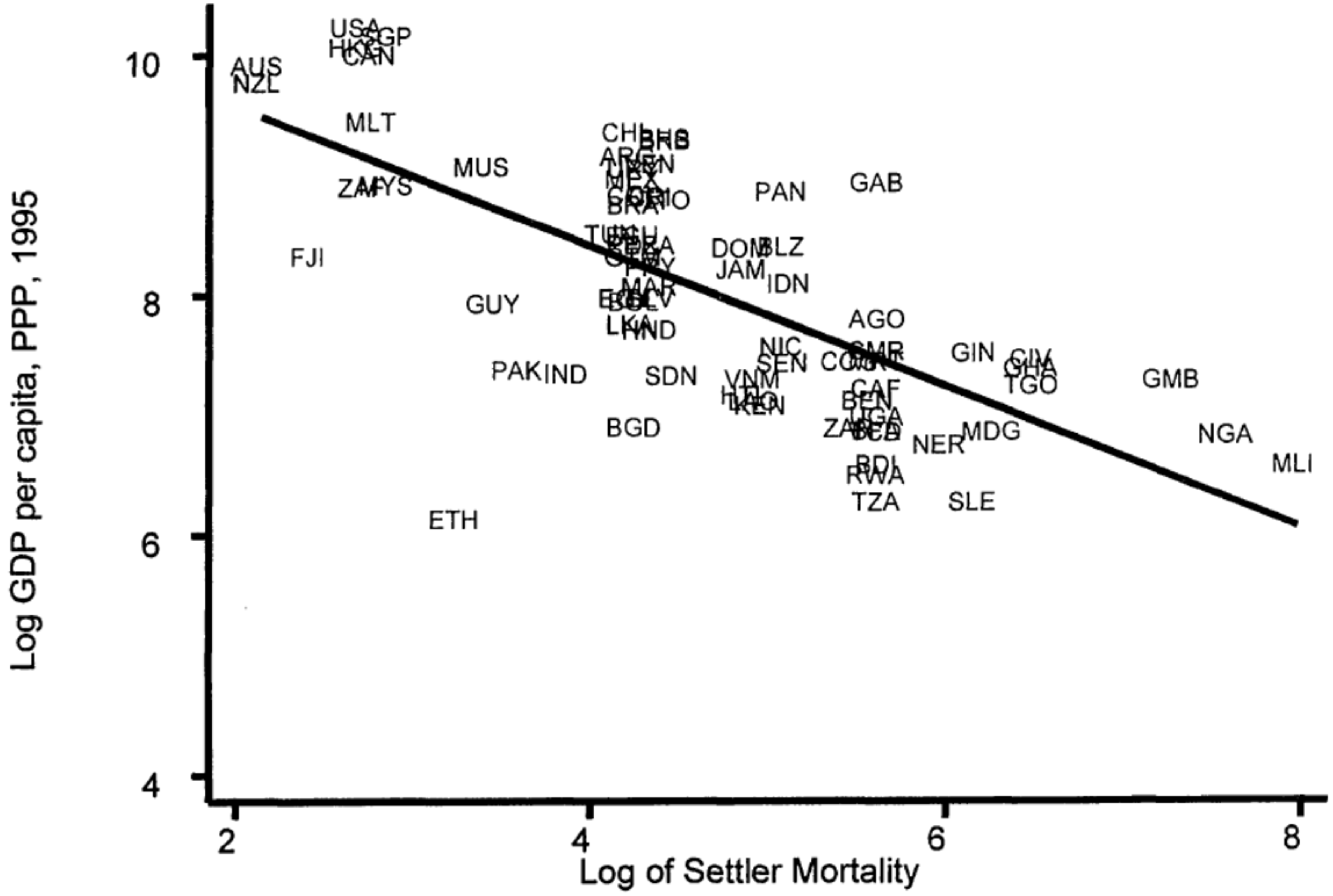


FIGURE 1. REDUCED-FORM RELATIONSHIP BETWEEN INCOME AND SETTLER MORTALITY

Another Example: AJR (2001) VI

Relationship Between X and Y

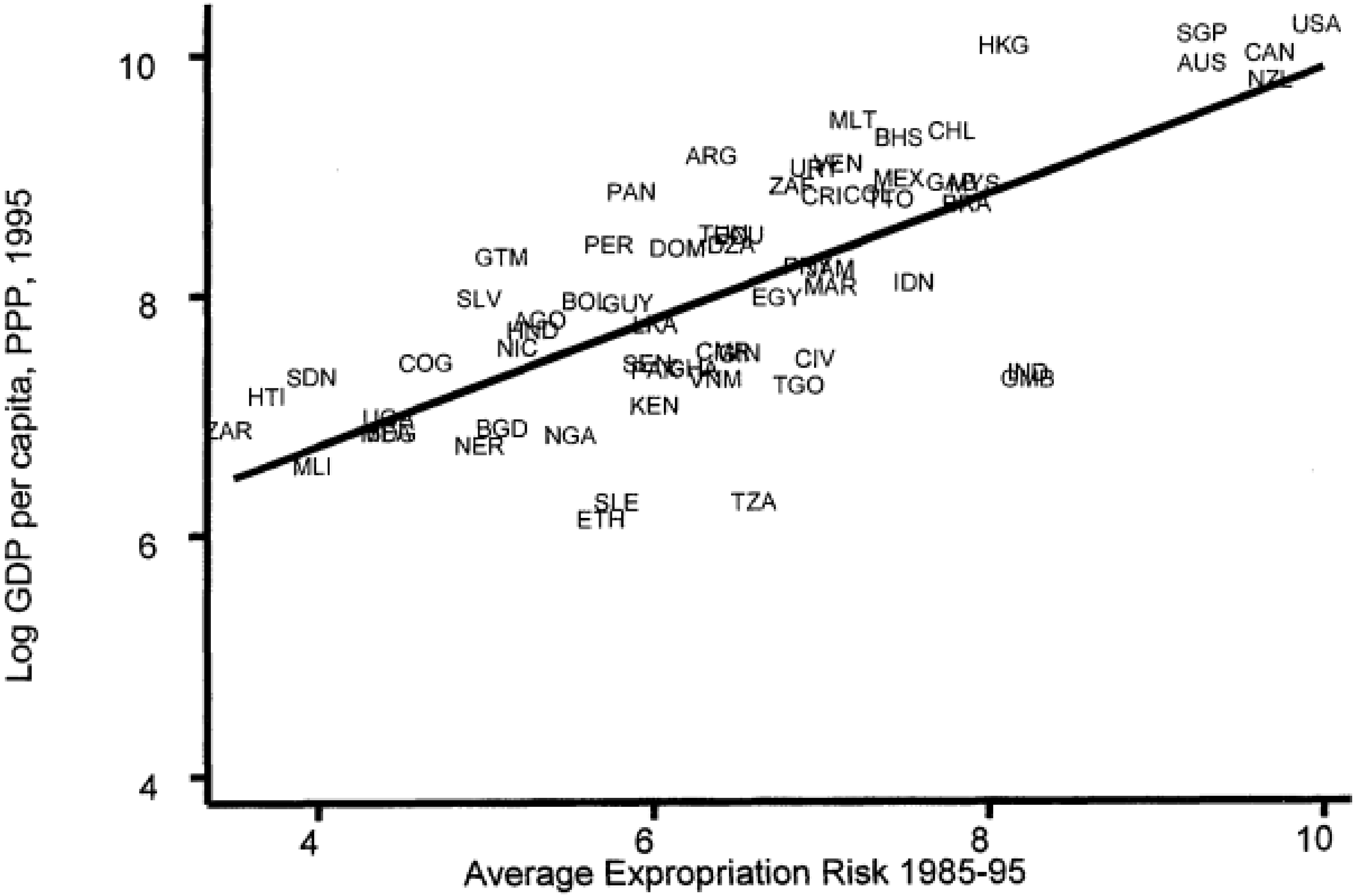


FIGURE 2. OLS RELATIONSHIP BETWEEN EXPROPRIATION RISK AND INCOME

Another Example: AJR (2001) VII

Relationship Between X and IV

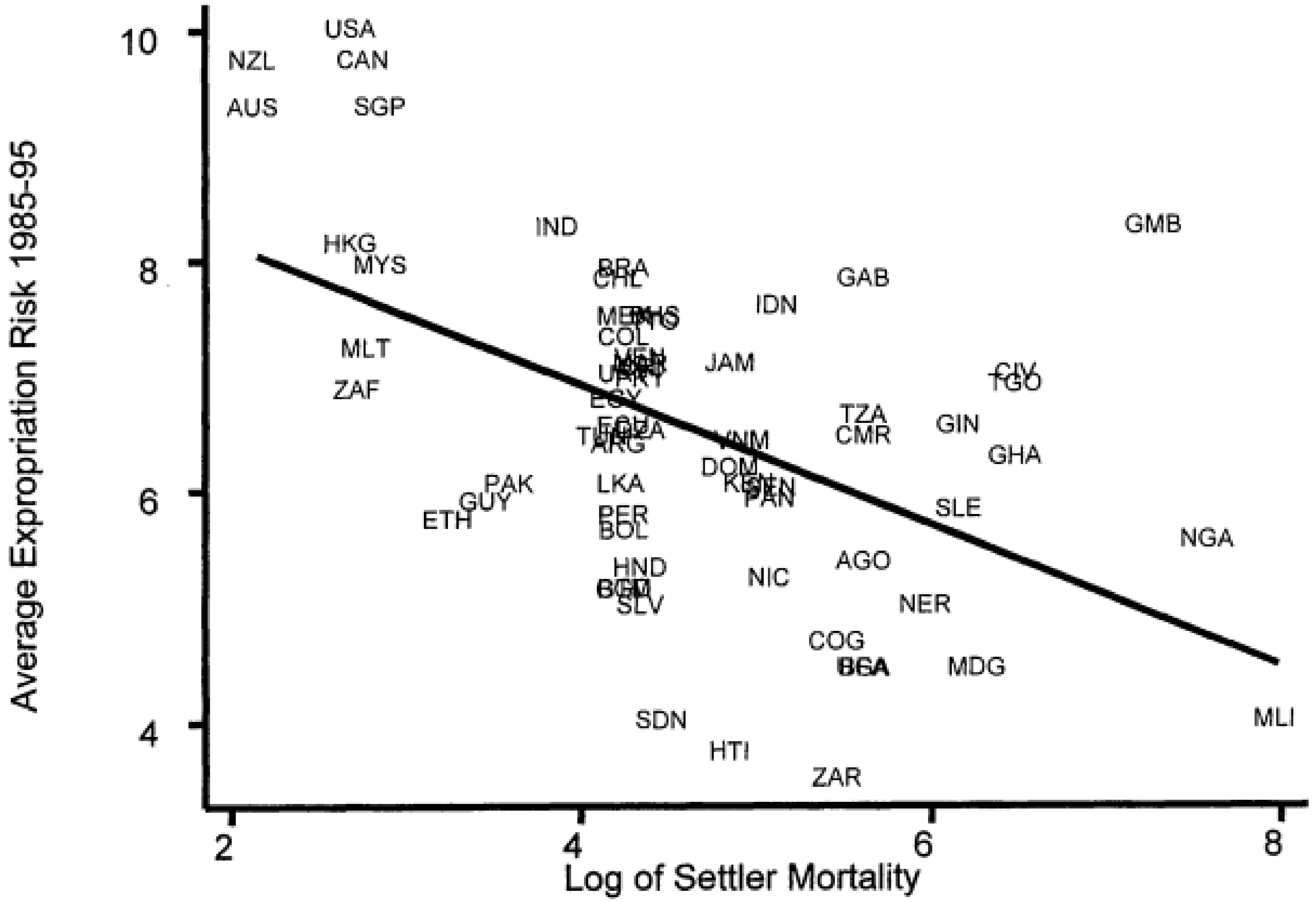


FIGURE 3. FIRST-STAGE RELATIONSHIP BETWEEN SETTLER MORTALITY AND EXPROPRIATION RISK

Another Example: AJR (2001) VIII

2SLS Results

TABLE 4—IV REGRESSIONS OF LOG GDP PER CAPITA

	Base sample (1)	Base sample (2)	Base sample without Neo-Europes (3)	Base sample without Neo-Europes (4)	Base sample without Africa (5)	Base sample without Africa (6)	Base sample with continent dummies (7)	Base sample with continent dummies (8)	Base sample, dependent variable is log output per worker (9)
Panel A: Two-Stage Least Squares									
Average protection against expropriation risk 1985–1995	0.94	1.00	1.28	1.21	0.58	0.58	0.98	1.10	0.98
	(0.16)	(0.22)	(0.36)	(0.35)	(0.10)	(0.12)	(0.30)	(0.46)	(0.17)
Latitude		−0.65		0.94		0.04		−1.20	
		(1.34)		(1.46)		(0.84)		(1.8)	
Asia dummy							−0.92	−1.10	
							(0.40)	(0.52)	
Africa dummy							−0.46	−0.44	
							(0.36)	(0.42)	
"Other" continent dummy							−0.94	−0.99	
							(0.85)	(1.0)	
Panel B: First Stage for Average Protection Against Expropriation Risk in 1985–1995									
Log European settler mortality	−0.61	−0.51	−0.39	−0.39	−1.20	−1.10	−0.43	−0.34	−0.63
	(0.13)	(0.14)	(0.13)	(0.14)	(0.22)	(0.24)	(0.17)	(0.18)	(0.13)
Latitude		2.00		−0.11		0.99		2.00	
		(1.34)		(1.50)		(1.43)		(1.40)	
Asia dummy							0.33	0.47	
							(0.49)	(0.50)	
Africa dummy							−0.27	−0.26	
							(0.41)	(0.41)	
"Other" continent dummy							1.24	1.1	
							(0.84)	(0.84)	
R^2	0.27	0.30	0.13	0.13	0.47	0.47	0.30	0.33	0.28
Panel C: Ordinary Least Squares									
Average protection against expropriation risk 1985–1995	0.52	0.47	0.49	0.47	0.48	0.47	0.42	0.40	0.46
	(0.06)	(0.06)	(0.08)	(0.07)	(0.07)	(0.07)	(0.06)	(0.06)	(0.06)
Number of observations	64	64	60	60	37	37	64	64	61

Simultaneous Causation & Structural Equation Modeling

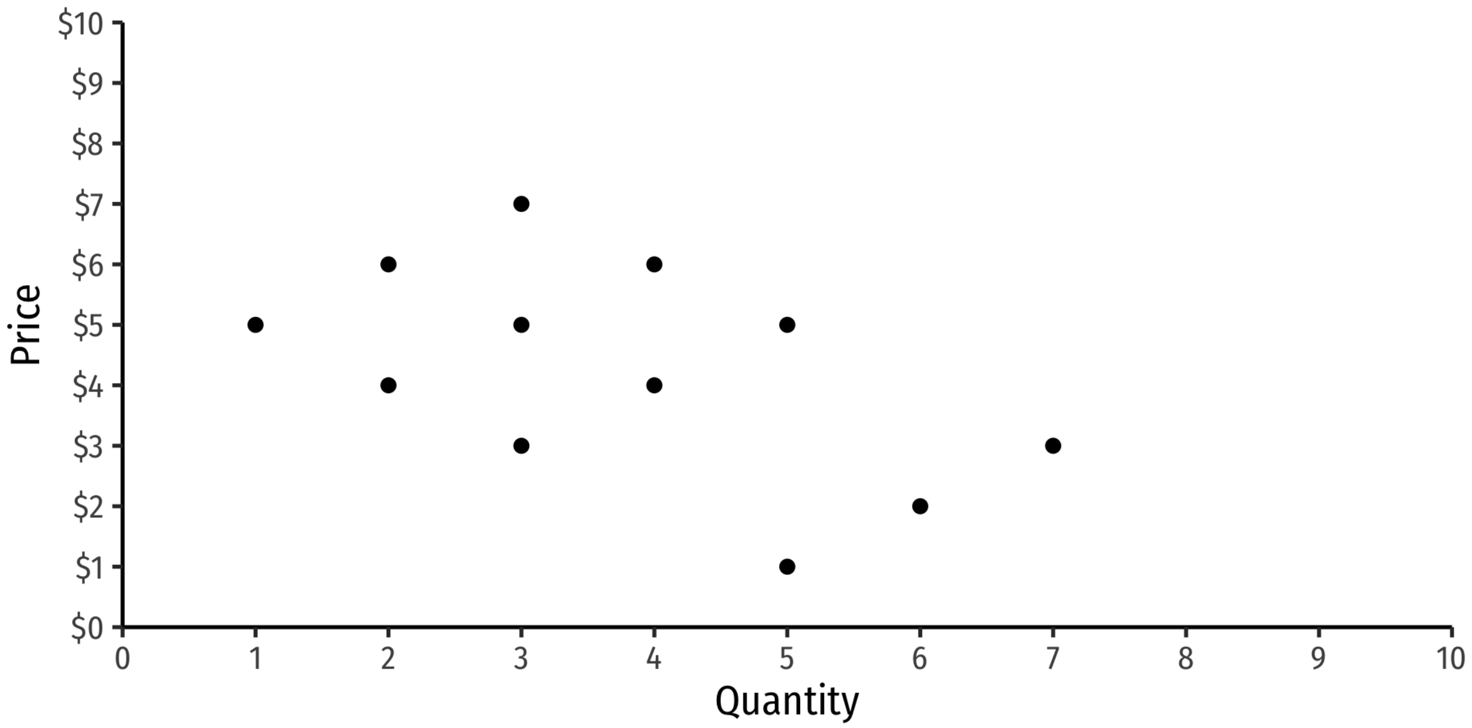
Simultaneous Causation

- Another classic use of instrumental variables in econometrics is to break through the problem of **simultaneous causation**

$$X \leftrightarrow Y$$

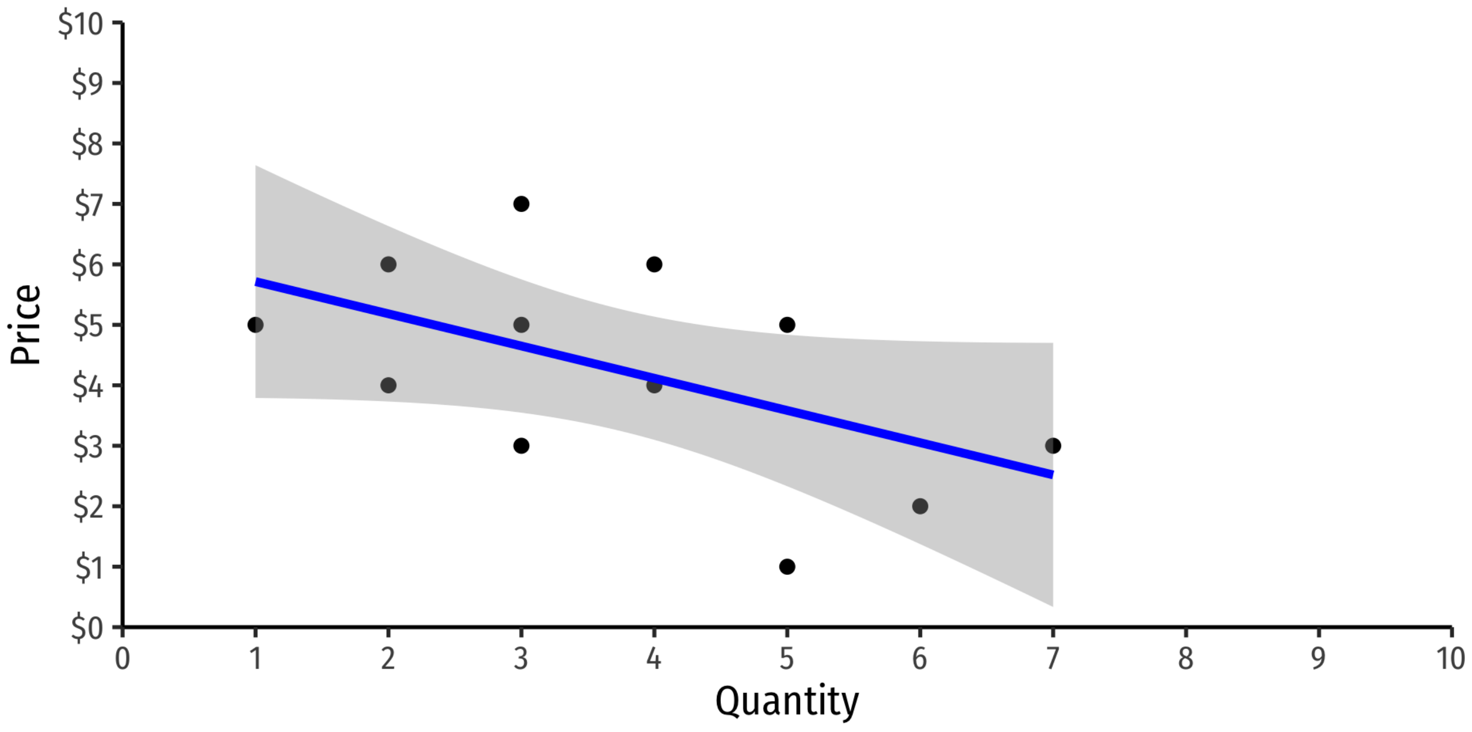
- This is a major source of endogeneity

Supply and Demand



- A famous example, foundational to our discipline, Supply and Demand
- Suppose you have data on price and quantity, and want to estimate a Demand curve with regression

Supply and Demand

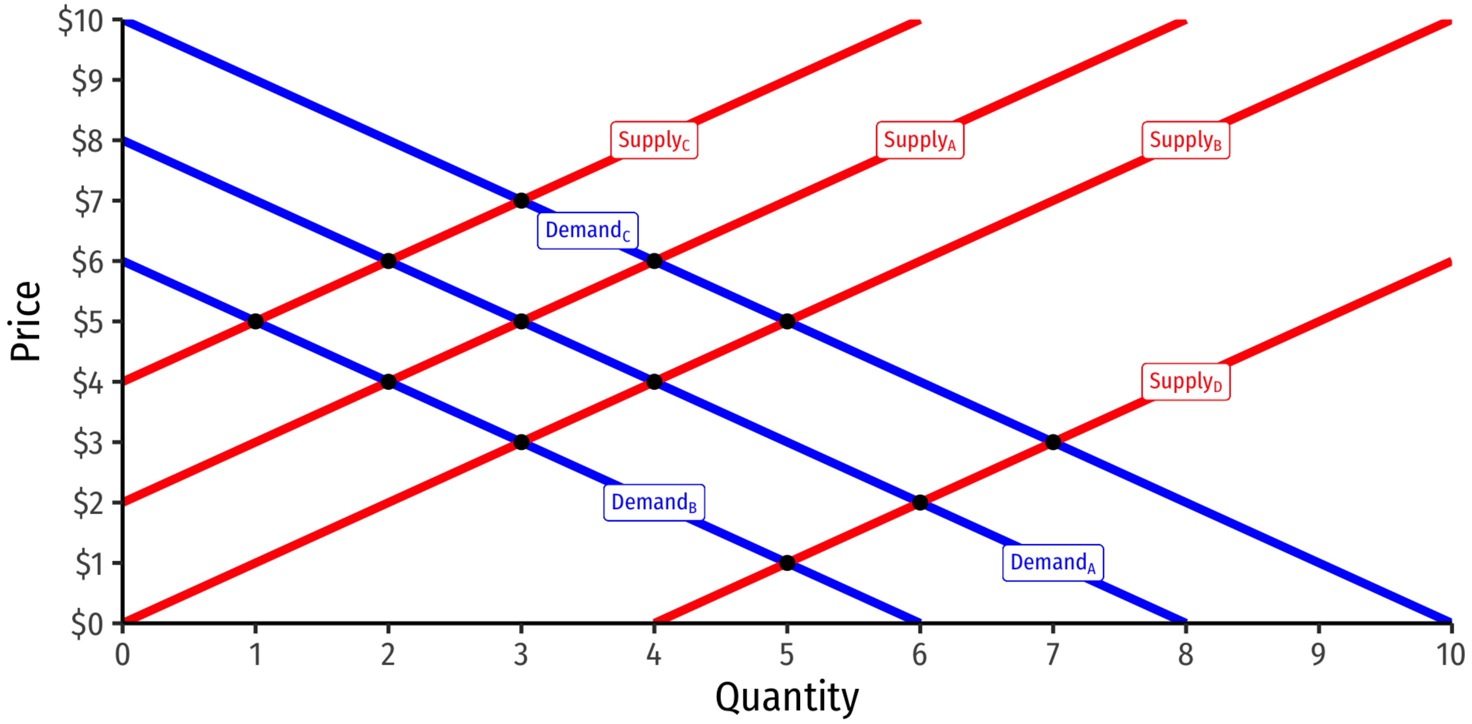


- A famous example, foundational to our discipline, Supply and Demand
- Suppose you have data on price and quantity, and want to estimate a Demand curve with regression
- Why can't we estimate the demand curve with a simple regression here?

$$\ln(\text{Quantity}_{it}) = \beta_0 + \beta_1 \ln(\text{Price}_{it}) + u_{it}$$

- With natural logs, β_1 is the **price elasticity of Demand**

Supply and Demand: Simultaneous Causality



- The data are actually all equilibrium (Q^*, P^*) points!
- Result of many demand and supply curve shifts & intersections!

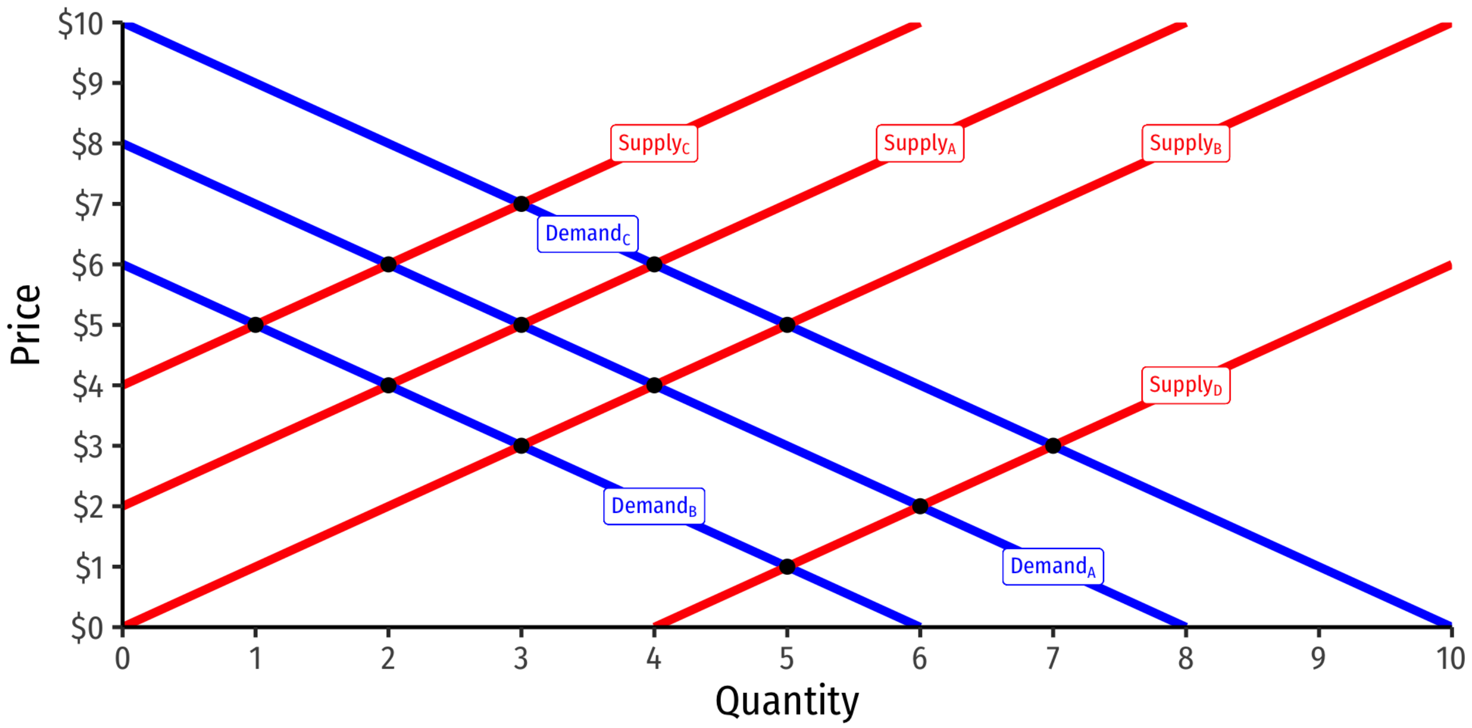
Supply and Demand: Simultaneous Causality

- **Structural equation model (SEM)** of demand and of supply:

$$Q_D = \alpha_0 + \alpha_1 P + \alpha_2 M + u_D$$
$$Q_S = \beta_0 + \beta_1 P + \beta_2 C + u_S$$

- α's and β's are parameters (to be estimated), u's are unobserved error terms
- P is price
  - Notice P **simultaneously determines** Q_D *and* Q_S!
- M are variables that shift demand (i.e. income, prices of other goods, etc)
- C are variables that shift supply (i.e. costs, etc)

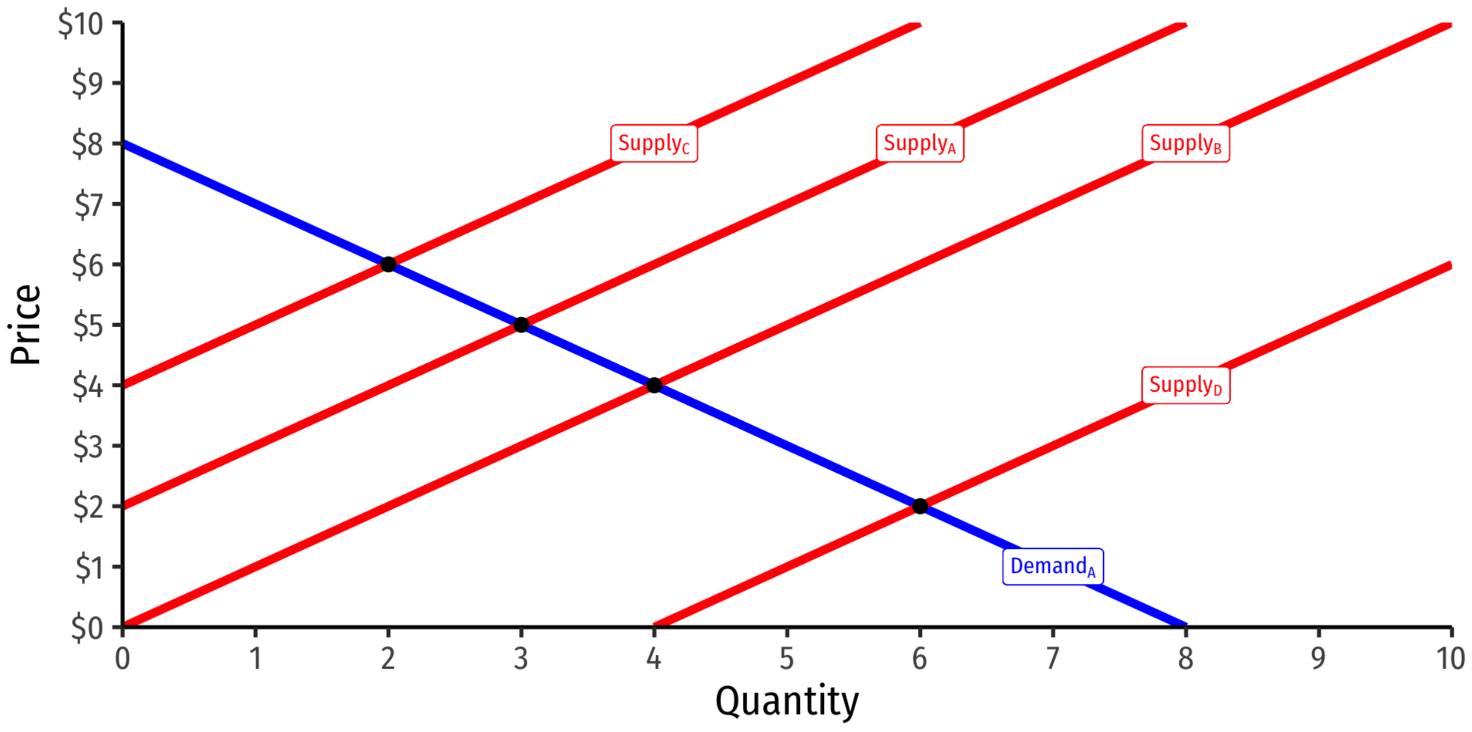
Supply and Demand: Simultaneous Causality



$$Q_D = \alpha_0 + \alpha_1 P + \alpha_2 M + u_D$$

- Why can't we just estimate price elasticity of demand (α_1) with the demand equation?
- P is partially a function of quantity supplied!

Supply and Demand: Simultaneous Causality



- [Instrumental variables] can identify the demand relationship
- Conceptually, use some supply shifter (like cost changes, C) correlated with price P, but not correlated with u_D
- Essentially: traces out unique demand relationship by allowing supply to vary & shift
- Then, can estimate demand elasticity β_1

Demand Example

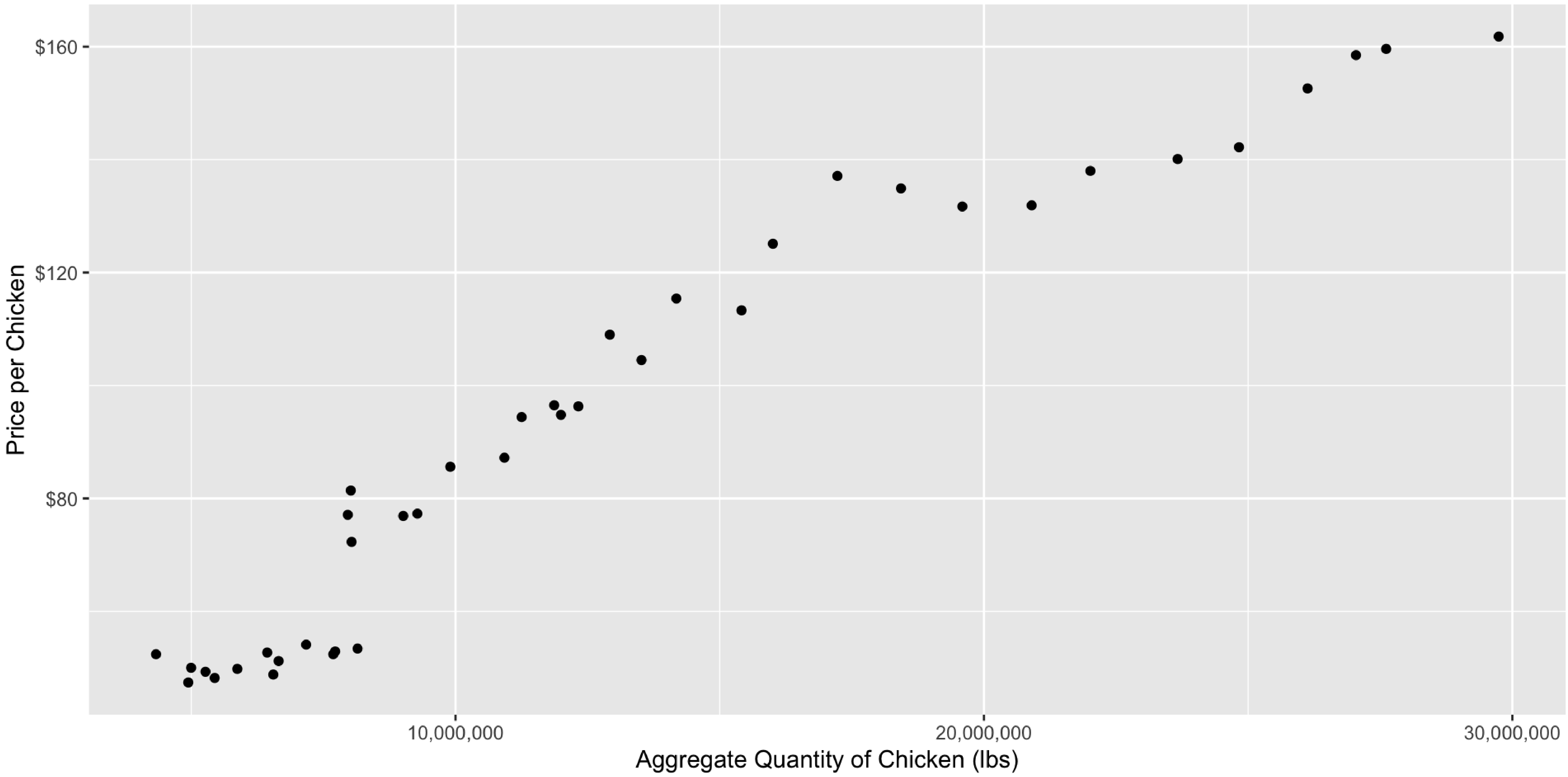
Example

Consider a famous the demand for broiler chickens 1960-1999

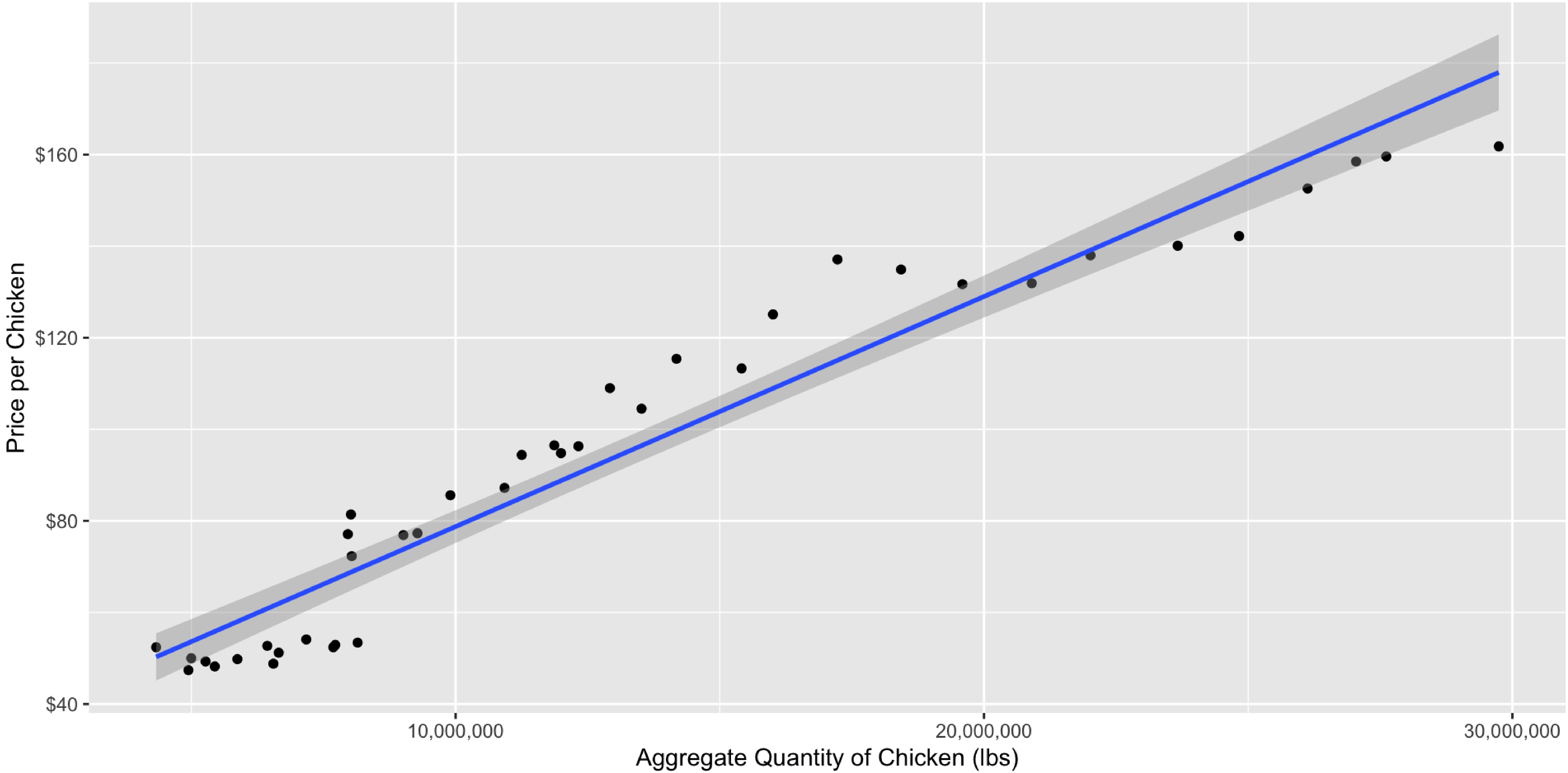
$$\ln \text{quantity}_t = \beta_0 + \beta_1 \ \ln \text{price of chicken}_t + \beta_2 \ \ln \text{price of beef}_t + \beta_3 \ \ln \text{population}_t + \beta_4 \ \ln \text{income}_t + u_t$$

Data from Epple, Dennis and Bennett T McCallum, 2006, "Simultaneous Equation Econometrics: The Missing Example," *Economic Inquiry* 44(2): 374-384

Demand Example



Demand Example 😨



Demand Example 😨

```
demand_reg <- lm(log_quantity ~ log_price, data = chick)
demand_reg %>% tidy()
```

term <chr>	estimate <dbl>	std.error <dbl>	statistic <dbl>
(Intercept)	10.624243	0.24379185	43.57916
log_price	1.258234	0.05445641	23.10535

2 rows | 1-4 of 5 columns

Demand Example With Controls

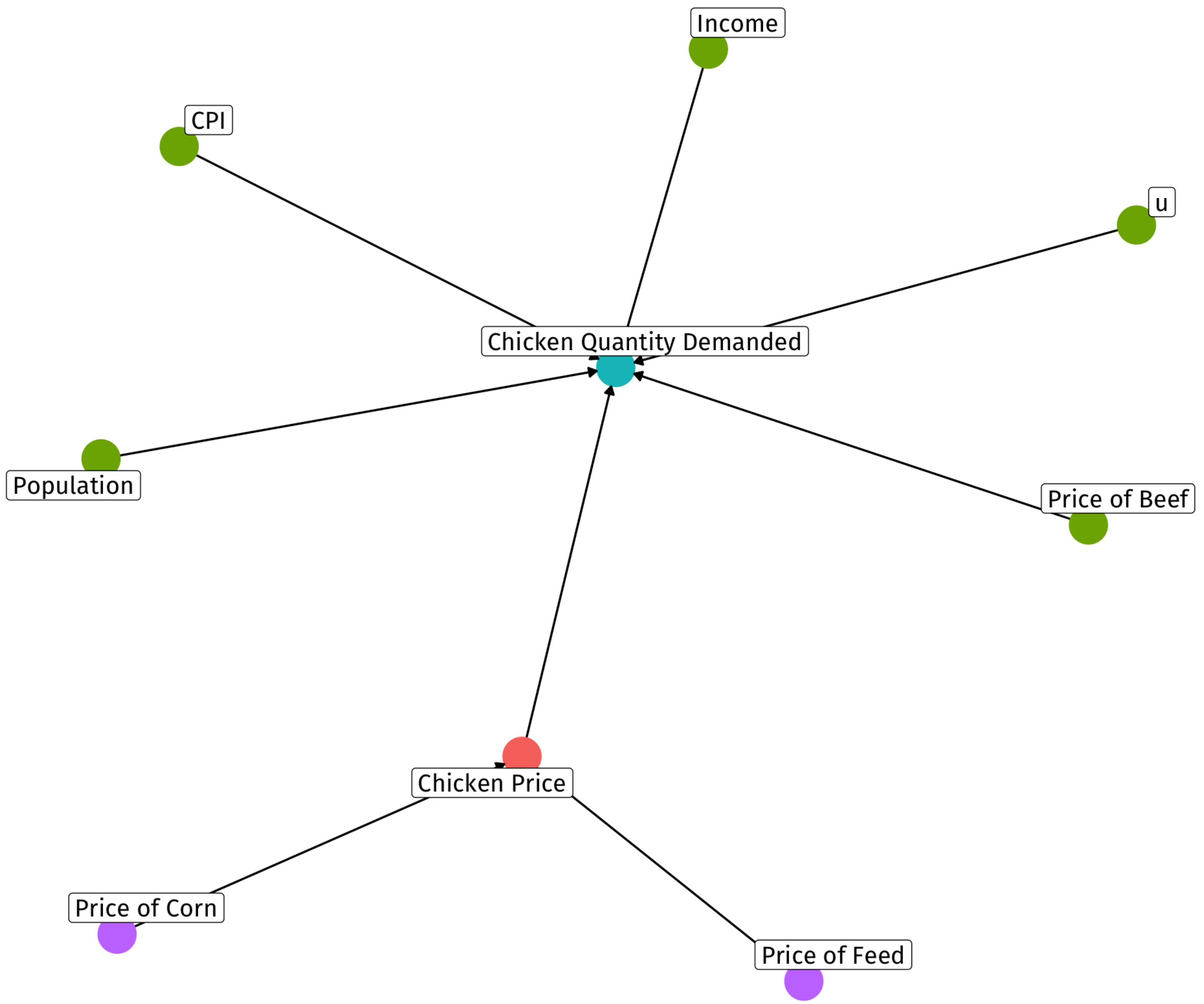
```
demand_reg_2 <- lm(log_quantity ~ log_price + log_income + log_beef + log_pop + CPI, data = chick)
demand_reg_2 %>% tidy()
```

term <chr>	estimate <dbl>	std.error <dbl>
(Intercept)	-3.292070075	1.5759805026
log_price	-0.280941924	0.0902419635
log_income	0.115668464	0.2169579441
log_beef	-0.048977631	0.0630734442
log_pop	3.592875888	0.5949973217
CPI	0.004718508	0.0009382211

6 rows | 1-3 of 5 columns

- $\hat{\beta}_1$: price elasticity of demand is -0.28%
- But this is biased! Endogeneity from simultaneous causation with supply *and* demand!

Consider the Causality



- Factors that influence quantity demanded:
  - price (endogenous! — partly determined by supply!)
  - price of substitutes (beef)
  - income
  - number of buyers (population)
  - price level (CPI)
  - other unobservables (u)
- Factors that influence price (on the supply side)
  - price of inputs/costs (feed and corn)
  - use these as **instruments** for price!

Instruments

- Use supply shifters, Price of Feed and Price of Corn (inputs/costs to raising chickens) as instruments for Chicken price
- Are they **relevant**? Check the first stage

```r
demand_first_stage <- lm(log_price ~ log_feed_price + log_corn_price + log_income + log_beef + log_pop + CPI, data = chick)
tidy(demand_first_stage)
```

term <chr>	estimate <dbl>	std.error <dbl>
(Intercept)	3.091394544	3.036679535
log_feed_price	0.291891404	0.145199951
log_corn_price	0.012500512	0.097284821
log_income	0.733391303	0.369170669
log_beef	-0.005344296	0.121075174
log_pop	-1.397997593	1.102062552
CPI	0.006140411	0.001825306

7 rows | 1-3 of 5 columns

```r
glance(demand_first_stage)
```

r.squared <dbl>	adj.r.squared <dbl>	sigma <dbl>	statistic <dbl>
0.9864229	0.9839544	0.0539911	399.5947

1 row | 1-4 of 12 columns

- `statistic` (F) is high enough, jointly significant
- Can also see correlations: ::: {.cell} ::: {.cell-output .cell-output-stdout}

```
              log_price log_feed_price log_corn_price
log_price      1.0000000      0.9464933      0.7742719
log_feed_price 0.9464933      1.0000000      0.9097980
log_corn_price 0.7742719      0.9097980      1.0000000
```

::: :::

Instruments

- Use supply shifters, Price of Feed and Price of Corn (inputs/costs to raising chickens) as instruments for Chicken price
- Are they **exogenous**?

$$\text{cor}(\text{Feed price}, u_D) = 0 \qquad \text{cor}(\text{Corn price}, u_D) = 0$$

- Argue that costs don't affect factors that affect demand (in error term); only affect supply

Second Stage

```
chick$price_hat <- demand_first_stage$fitted.values
```

Now regress quantity on the fitted values of $\widehat{price}$ (from first stage) with all the covariates (from first stage)

```
demand_second_stage <- lm(log_quantity ~ price_hat + log_income + log_beef + log_pop + CPI, data = chick)
tidy(demand_second_stage)
```

term <chr>	estimate <dbl>	std.error <dbl>
(Intercept)	-2.686673617	1.690775142
price_hat	-0.437551695	0.158858181
log_income	0.209226904	0.235386608
log_beef	0.004414143	0.078219995
log_pop	3.388282917	0.632782246
CPI	0.005537623	0.001175213

6 rows | 1-3 of 5 columns

```
glance(demand_second_stage)
```

r.squared <dbl>	adj.r.squared <dbl>	sigma <dbl>	statistic <dbl>
0.9964764	0.9959582	0.03528781	1923.027

1 row | 1-4 of 12 columns

Using fixest

```
iv_demand_reg <- feols(log_quantity ~ log_income + log_beef + log_pop + CPI | log_price ~ log_feed_price + log_corn_price, data = chick)
iv_demand_reg
```

```
TSLS estimation, Dep. Var.: log_quantity, Endo.: log_price, Instr.: log_feed_price, log_corn_price
Second stage: Dep. Var.: log_quantity
Observations: 40
Standard-errors: IID
               Estimate Std. Error   t value   Pr(>|t|)
(Intercept)   -2.686674   1.721040 -1.561075 1.2777e-01
fit_log_price -0.437552   0.161702 -2.705918 1.0572e-02 *
log_income     0.209227   0.239600  0.873234 3.8866e-01
log_beef       0.004414   0.079620  0.055440 9.5611e-01
log_pop        3.388283   0.644109  5.260417 7.8866e-06 ***
CPI            0.005538   0.001196  4.629155 5.1702e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 0.033116   Adj. R2: 0.995812
F-test (1st stage), log_price: stat = 8.46361, p = 0.001079, on 2 and 33 DoF.
                  Wu-Hausman: stat = 1.57081, p = 0.218897, on 1 and 33 DoF.
                      Sargan: stat = 2.88552, p = 0.089379, on 1 DoF.
```

Comparing

	OLS	OLS	2SLS (by hand)	2SLS (fixest)
Constant	10.624***	-3.292**	-2.687	-2.687
	(0.244)	(1.576)	(1.691)	(1.721)
Log Price/lb of Chicken	1.258***	-0.281***	-0.438***	-0.438**
	(0.054)	(0.090)	(0.159)	(0.162)
Log Income		0.116	0.209	0.209
		(0.217)	(0.235)	(0.240)
Log Price/lb of Beef		-0.049	0.004	0.004
		(0.063)	(0.078)	(0.080)
Log Population		3.593***	3.388***	3.388***
		(0.595)	(0.633)	(0.644)
CPI		0.005***	0.006***	0.006***
		(0.001)	(0.001)	(0.001)
n	40	40	40	40
Adj. R^2	0.93	1.00	1.00	1.00
SER	0.14	0.03	0.03	0.03

* p < 0.1, ** p < 0.05, *** p < 0.01

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